Exploding Dots™ Teaching Guide

Experience 5:

Division

Related resources:

- Access videos of *Exploding Dots™* lessons at: http://gdaymath.com/courses/exploding-dots/
- Be sure to review the *Getting Started* guide, available here.
- Printable student handouts for this experience are available here.

Experience 5: Division
Overview

Student Objectives

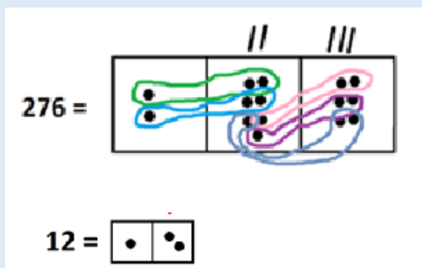
Division can be interpreted as a process of counting: the computation 276 ÷ 12 is simply asking for the number of groups of 12 one can find in a picture of 276. The visuals of dots-and-boxes make this counting process extraordinarily natural.

The Experience in a Nutshell

To compute 276 ÷ 12 we could draw a picture of 276 dots on a page and then circle groups of twelve dots. The number of groups of twelve we find is the answer to the division problem.

But this is a highly inefficient way to compute division!

Alternatively, we could draw a picture of 276 in a 1 ← 10 machine and look for groups of twelve in that picture. We readily see that there are two groups at the tens level and three at the ones level, that is, that there are 23 groups in all. We have 276 ÷ 12 = 23.



Remainders are readily identified too.

$$277 \div 12 = 23 \quad R\,1$$

Setting the Scene

View the welcome video from James to set the scene for this experience:
http://gdaymath.com/lessons/explodingdots/5-1-welcome/ [1:43 minutes]

Division

This is Core Lesson #11, corresponding to Lesson 5.2 on gdaymath.com/courses/exploding-dots/.

James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/5-2-division/ [7:38 minutes]

Here is the script James follows when he gives this lesson on a board. Of course, feel free to adapt this wording as suits you best. You will see in the video when and how James draws the diagrams and adds to them.

Addition, subtraction, multiplication. Now it is time for division.

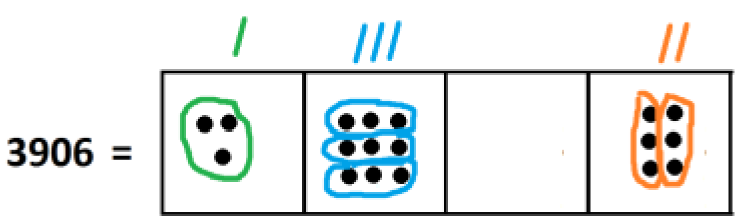
Let's start slowly with a division problem whose answer we might be able to see right away.

What is 3906 ÷ 3*?*

The answer is 1302.

If you think of 3906 as 3000 + 900 + 6, then we can see that dividing by three then gives 1000 + 300 + 2.

And we can really see this if we draw a picture of 3906 in a 1 ← 10 machine. We see groups of three: 1 group at the thousands level, 3 groups at the hundreds level, and 2 groups at the ones level.

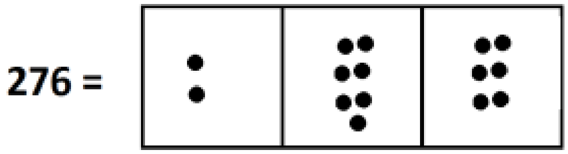


That's it! We're doing division and seeing the division answers just pop right out!

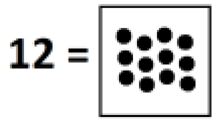
Division by single-digit numbers is all well and good. What about division by multi-digit numbers? People usually call that *long division*.

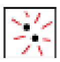
Let's consider the problem 276 ÷ 12.

Here is a picture of 276 in a 1 ← 10 machine.

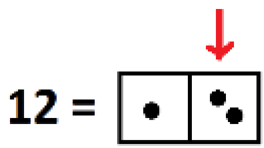


And we are looking for groups of twelve in this picture of 276. Here's what twelve looks like.



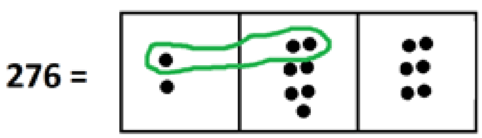


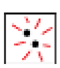
Actually, this is not right as there would be an explosion in our 1 ← 10 machine. Twelve will look like one dot next to two dots. But we need to always keep in mind that this really is a picture with all twelve dots residing in the rightmost box.



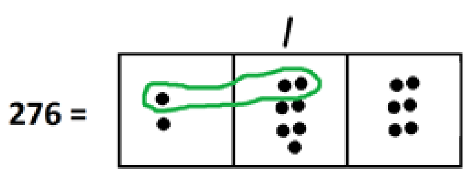
Okay. So, we're looking for groups of 12 in our picture of 276. Do we see any one-dot-next-to-two-dots in the diagram?

Yes. Here's one.



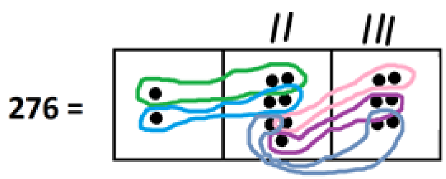


Within each loop of 12 we find, the 12 dots actually reside in the right part of the loop. So, we have found one group of 12 at the tens level.



This point is subtle: the twelve dots of the loop are sitting in the right part of the loop. One might have to remind students a few times as the division process is conducted. It helps to ask: If you were to unexplode some dots just within the loop, which dots could unexplode (and stay in the loop) and in which box would all twelve dots finally be?

And there are more groups of twelve.



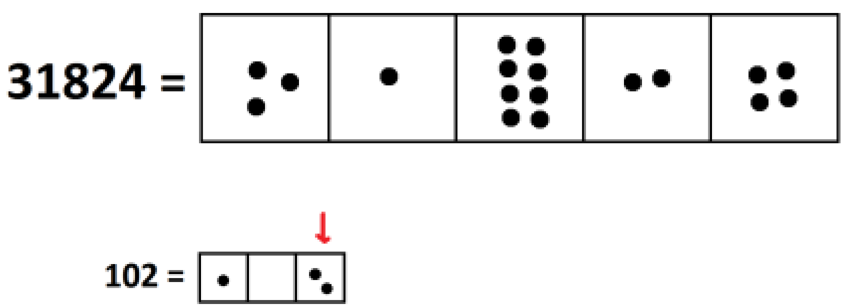
We see a total of two groups of 12 at the tens level and three 12's at the ones level. The answer to 276 ÷ 12 is thus, 23.

BY THE WAY … I LEAVE THE PICTURE OF 276 ÷ 12 ON THE BOARD! THIS IS IMPORTANT IF YOU ARE GOING TO THE SECTION ON POLYNOMIALS IN EXPERIENCE 6. I am surreptitious about this, just making it look as though I just don't happen to erase that part of the board, even as we go on to more examples and more work.

At this point I might ask students to try one "on their ownsies": 2783 ÷ 23 and then 31824 ÷ 102 and mention that this second one "has a hiccup." Or, I might just do 31824 ÷ 102 next as part of the lecture. It all depends on what I feel is appropriate for the students.

Let's do another example. Let's compute 31824 ÷ 102.

Here's the picture.



Now we are looking for groups of one dot–no dots–two dots in our picture of 31824. (And, remember, all 102 dots are physically sitting in the rightmost position of each set we identify.) This is the same subtle point as before.

We can spot a number of these groups. (I now find drawing loops messy so I am drawing Xs and circles and boxes instead. Is that okay? Do you also see how I circled a double group in one hit at the very end?)

31824 =

The answer 312 to 31824 ÷ 102 is now apparent.

I do usually take the time to explain how the dots-and-boxes method of division aligns with the traditional algorithm. It is tricky for me to write about this here as different countries have different notational systems for long division and slightly different approaches to the algorithm. For example, in the U.S. it has become customary to think of long division as "repeated subtraction," whereas in Serbia and Australia, for example, a more mysterious algorithm is taught.

Watch the video of lesson 5.6 here http://gdaymath.com/lessons/explodingdots/5-6-remainders/ to see the Australian approach and what I usually do for students right at this moment, and see the text that follows that video in Lesson 5.6 on the site for the U.S. approach. For your own country's approach, I am afraid to say it is up to you to figure out the connection to dots-and-boxes (it is likely to be very similar), and to decide if you want to share it with your students at this point.

Remainders

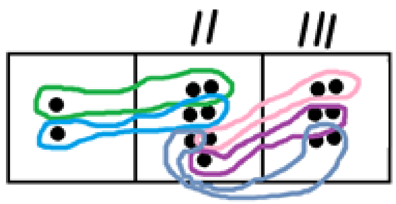
This is Core Lesson #12, corresponding to Lesson 5.4 on gdaymath.com/courses/exploding-dots/.

James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/5-4-remainders/ [1:59 minutes]

We saw that 276 ÷ 12 equals 23.

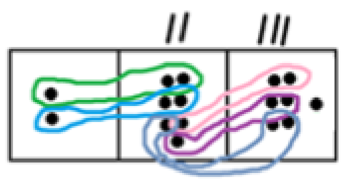
276 =



Suppose we tried to compute 277 ÷ 12 instead. What picture would we get? How should we interpret the picture?

Well, we'd see the same picture as before except for the appearance of one extra dot, which we fail to include in a group of twelve.

277 =



This shows that 277 ÷ 12 equals 23 with a remainder of 1.

You might write this as

$$277 \div 12 = 23\ R\ 1$$

or with some equivalent notation for remainders. (People use different notations for remainders in different countries.)

Or you might be a bit more mathematically precise and say that 277 ÷ 12 equals 23 with one more dot still to be divided by twelve:

$$277 \div 12 = 23 + \frac{1}{12}.$$

If you alter the picture of 276 ÷ 12 on the board by adding a dot to make 277 ÷ 12, be sure to casually erase the extra dot and return the picture to 276 ÷ 12. We need this picture for when we move to work on polynomials in Experience 6.

Handout A: *Division and Remainders*

Use the student handout shown below for students who want practice questions from this lesson to mull on later at home. This is NOT homework; it is entirely optional. (See the document "Experience 5: Handouts" for a printable version.)

Exploding Dots

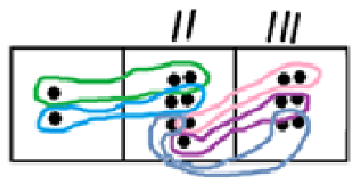
Experience 5: Division

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

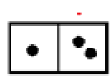
Handout A: *Division and Remainders*

This picture shows that 276 ÷ 12 equals 23.

276 =



12 =

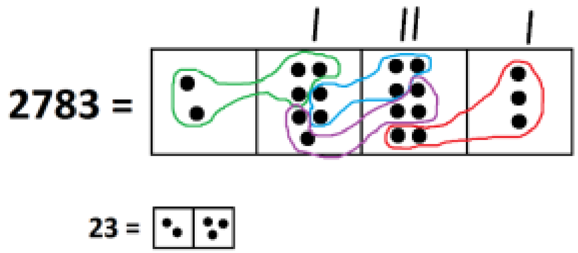


Here are some practice questions you might, or might not, want to try.

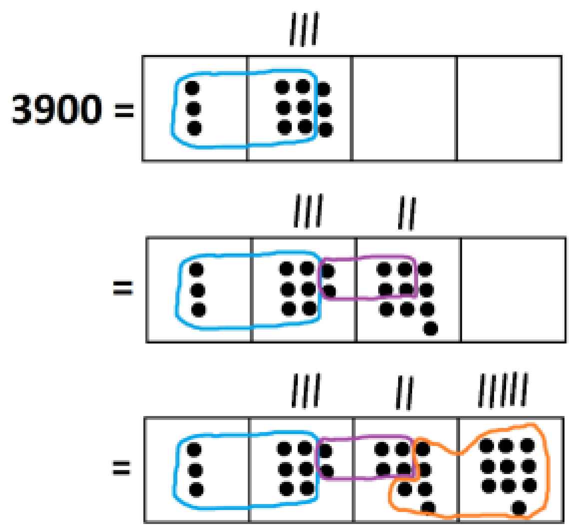
1. Compute 2783 ÷ 23 by the dots-and-boxes approach by hand.
2. Compute 3900 ÷ 12.
3. Compute 46632 ÷ 201.
4. Show that 31533 ÷ 101 equals 312 with a remainder of 21.
5. Compute 2789 ÷ 11.
6. Compute 4366 ÷ 14.
7. Compute 5481 ÷ 131.
8. Compute 61230 ÷ 5.

Solutions to Handout A

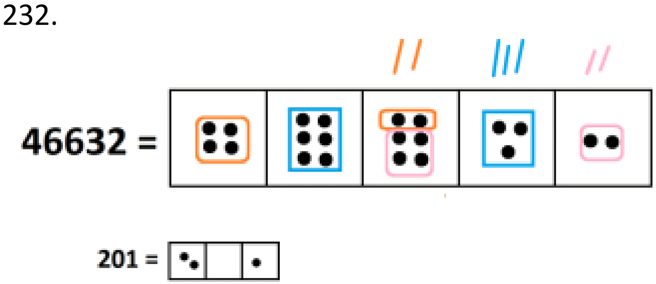
1. 2783 ÷ 23 = 121



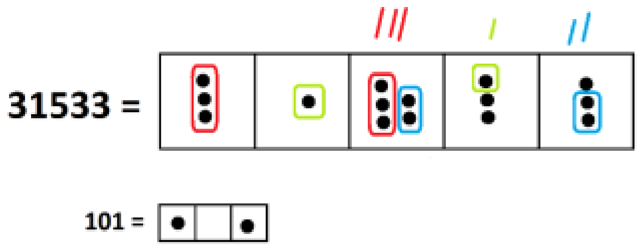
2. 3900 ÷ 12 = 325. We need some unexplosions along the way. (And can you see how I am getting efficient with my loop drawing?)



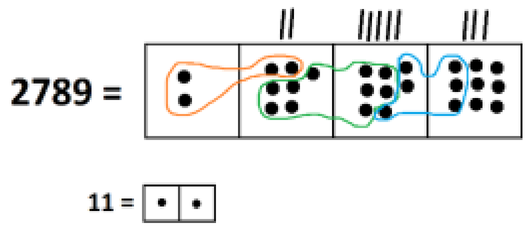
3. 46632 ÷ 201 = 232.



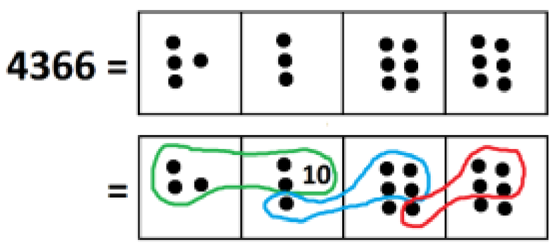
4. $31533 \div 101 = 312$ with a remainder of 21. That is, $31533 \div 101 = 312 + \frac{21}{101}$



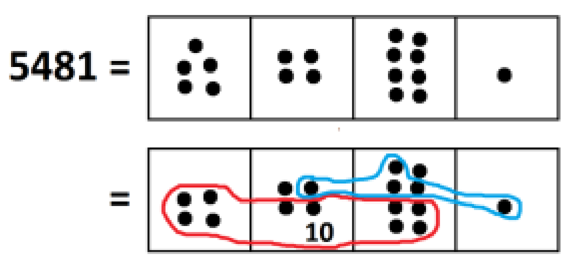
5. We have $2789 \div 11 = 253$ with a remainder of 6. That is, $2789 \div 11 = 253 + \frac{6}{11}$.



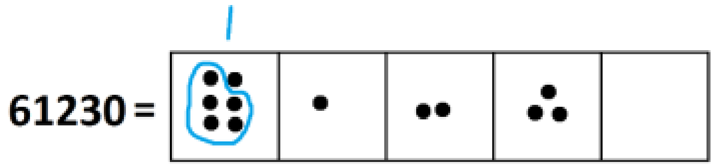
6. $4366 \div 14 = 311 + \frac{12}{14}$.



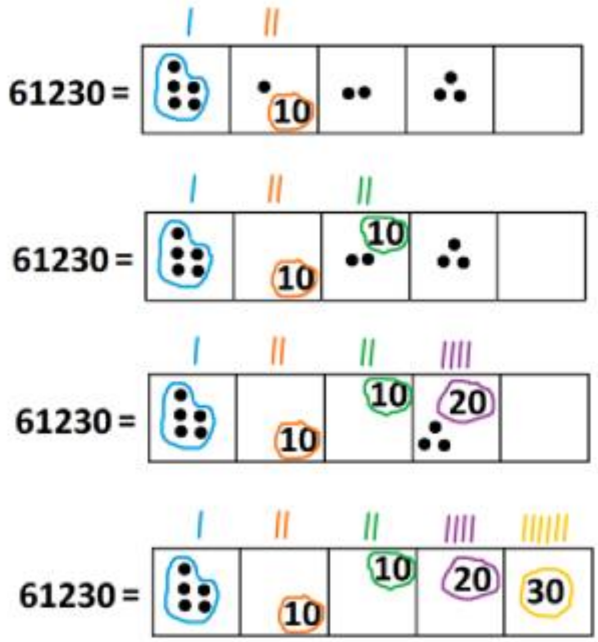
7. $5481 \div 131 = 41 + \frac{110}{131}$.



8. We certainly see one group of five right away.



Let's perform some unexplosions. (And let's write numbers rather than draw lots of dots. Drawing dots gets tedious!)



We see $61230 \div 5 = 12246$.

Handout B: *Wild Explorations*

Use the student handout shown below for students who want some deep-thinking questions from this Experience to mull on later at home. This is NOT homework; it is entirely optional, but this could be a source for student projects. (See the document "Experience 5: Handouts" for a printable version.)

Exploding Dots

Experience 5: Division

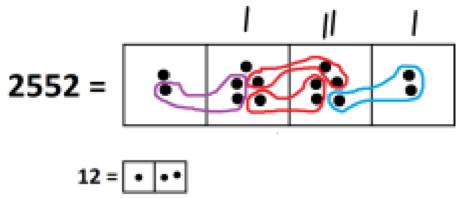
Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout B: *WILD EXPLORATIONS*

Here is a "big question" investigation you might want to explore, or just think about. Have fun!

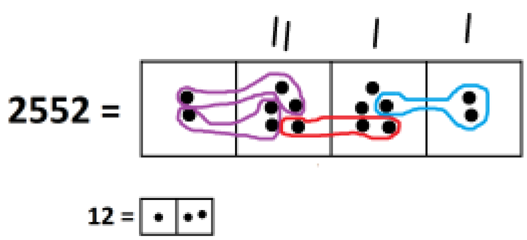
EXPLORATION: LEFT TO RIGHT? RIGHT TO LEFT? ANY ORDER?

When asked to compute $2552 \div 12$, Kaleb drew this picture, which he got from identifying groups of twelve working right to left.



He said the answer to $2552 \div 12$ is 121 with a remainder of 1100.

Mabel, on the other hand, identified groups of twelve from left to right in her diagram for the problem.



She concluded that $2552 \div 12$ is 211 with a remainder of 20. Both Kaleb and Mabel are mathematically correct, but their teacher pointed out that most people would expect an answer with smaller remainders: both 1100 and 20 would likely be considered strange remainders for a problem about division by twelve. She also showed Kaleb and Mabel the answer to the problem that is printed in the textbook.

$$2552 \div 12 = 212 \;\; R\; 8$$

How could Kaleb and Mabel each continue work on their diagrams to have this textbook answer appear?