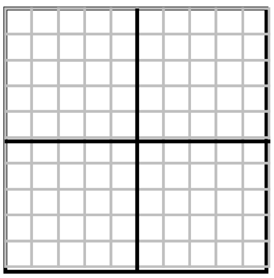
**AREA MODELS link spatial reasoning to learning mathematics.**
Deliberately infusing thinking, reasoning and problem solving into teaching the BASICS Grades 2 to 12.

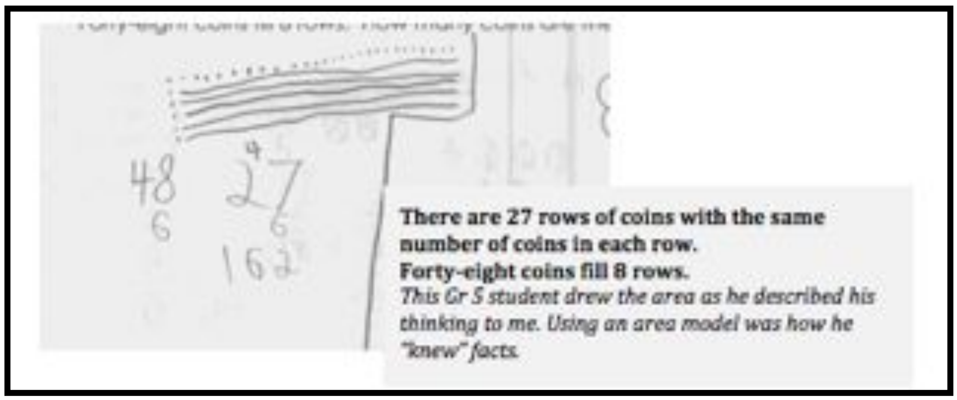
*The relation between spatial ability and mathematics is so well established that it no longer makes sense to ask whether they are related. (Mix & Cheng, 2012).*

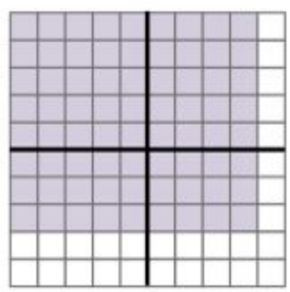
Area Models build conceptual understanding through spatial reasoning as they engage actively students in thinking and problem solving while they "learn" basics.

*Conceptual understanding in mathematics means that students understand which ideas are key (by being helped to draw inferences about those ideas) and that they grasp the heuristic value of those ideas. They are thus better able to use them strategically to solve problems – and to avoid common misunderstandings. -Grant Wiggins*



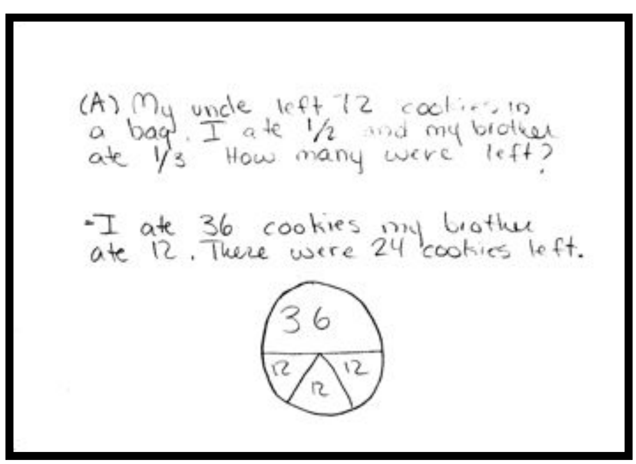
Gr 2 student: To solve 47 + 39 I thought it in my head. I see 47 and 3 more to 50, then 30 to 80 and 6. That's 86. But you could go 39 to 40 and add 46



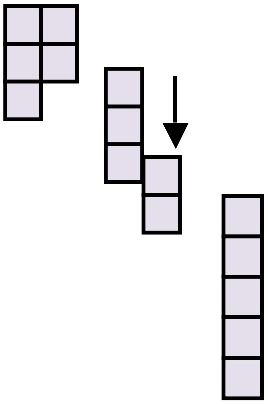



*Do you see 9 x 8?*
*You can think (5+4) 8 or 9 (5+3).*
*So 40 + 32 or 45 + 27 which I do not like as much. Either way 9 x 8 = 72.*
*Grade 4 student*

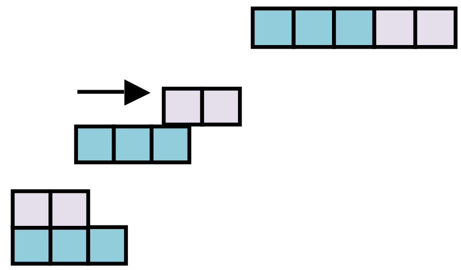
*These materials deliberately focus first on the use of **rectangular arrays** , using imagery that focuses on 2 dimensions and square units to support a view that agrees "the most flexible and robust interpretation of multiplication is based on a rectangle" (Davis, 2008, p. 88). Because students tend to hold on to the representations they are initially exposed to as the grounding for their conceptual understanding (Pirie and Kieran, 1994) The use of "dots" and bingo chips to build arrays as well as the use of "pizza" models for **introducing and developing early ideas** about multiplication and fractions are noticeably absent in the instructional materials. However, as this example demonstrates the use of rectangular models is not proceduralized.*



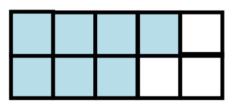
FIVE FRAMES AND TEN FRAMES are the first "AREA" Models I use. *See DOTS, RODS and CHUNKZ for INFORMATION and background on developing Relational Thinking and Fluency with Number to 20.*



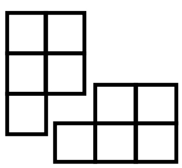
*Five as 2 and 3 translates to 5 in a line. The most difficult way to trust a small set is when it is arranged in a line.*
*The subitize sets of 3 and 2 are easily discerned when we use colour.*
*But with practice, the goal is to trust 5 :*
*Automatic recognition of arrangements.*

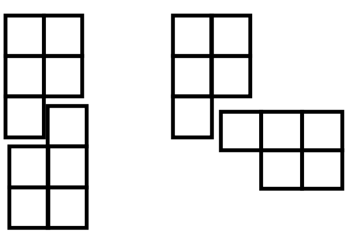


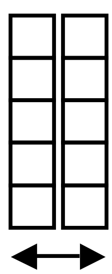
| | | | |
|---|---|---|---|
| SUBITIZE | CONSERVE | CORRESPOND | COMPOSE/DECOMPOSE |
| TRANSLATE | TRANSFORM | VISUALIZE | |
| PARTITION | ITERATE | UNITIZE | |

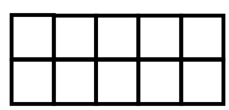


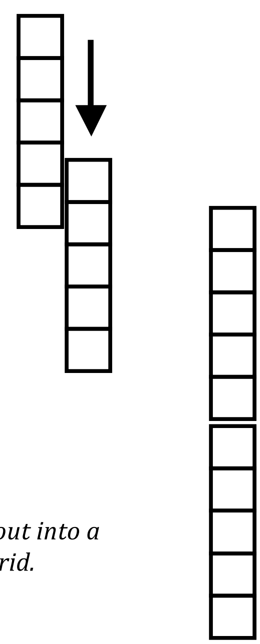
*I see 4 + 3 = 7*
*I see 7 + 3 = 10*
*I see 10 – 7 = 3*
*What do you see?*











*Two fives become ten.*
*The possibilities multiply.*
*You can "fit" the five CHUNKZ together.*
*You can place the 2 fives side by side.*
*Both are valuable ways to "see" the ten.*
*THE GOAL: trust 10*
*Practice decomposing numbers by relating to 10.*
*(Learn number "facts".)*

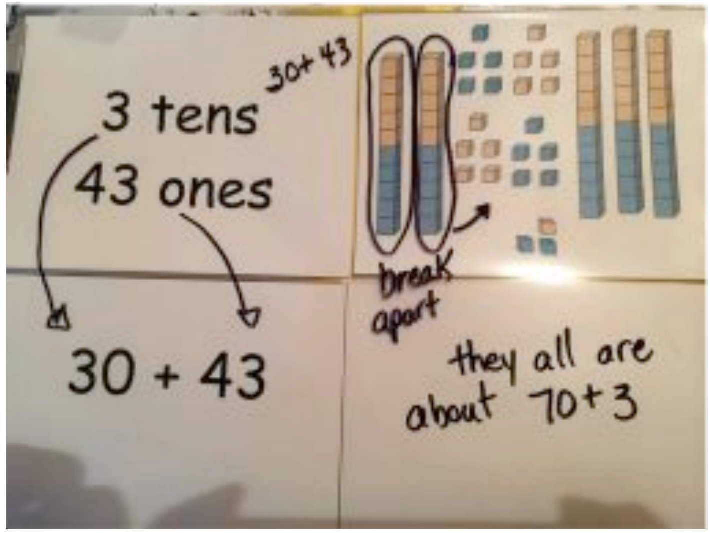
**Alberta Curriculum:**
**Kindergarten, Grade 1, 2**

*The ten frame eventually stretches out into a long ten to be fitted into a 10 x 10 grid.*

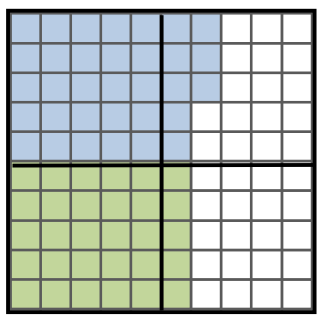
**TRANSLATE the BUILDS** by connecting area models to linear models.
**VISUALIZE Relationships**

Students work with BERCS cards to BUILD EXPLAIN REPRESENT COMPARE numbers

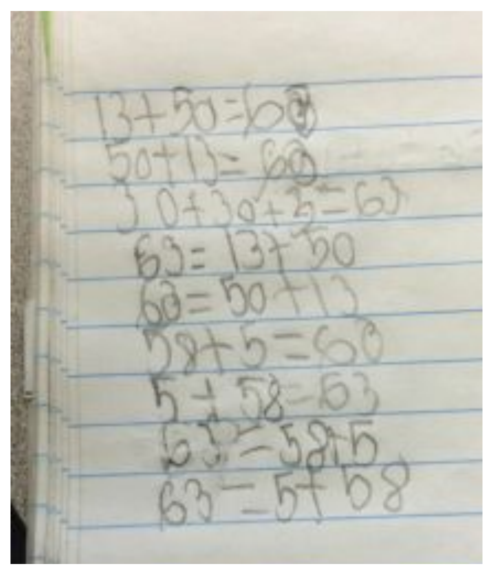
Organizing, ordering, recording the results.
Searching patterns , describing generalizations

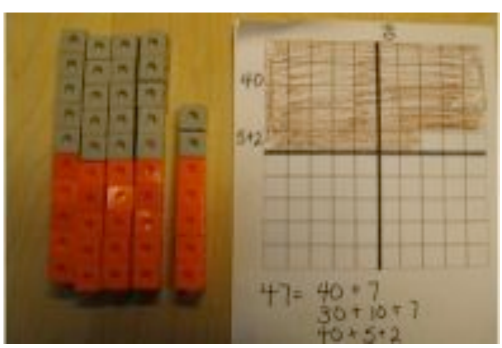



**Organize the BUILD by orienting** into a 10 by 10 area grid. **25, 50, 75 and 100 are referents.**
**VISUALIZE ESTIMATE REASON**



*Can you "see" his equations in the grid for 63?*



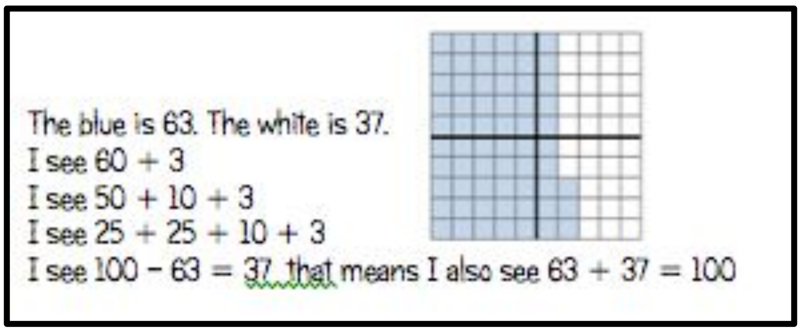

**Describe the Number**
Students compose and decompose 2 digit numbers moving between the "frame", the numberline and real materials

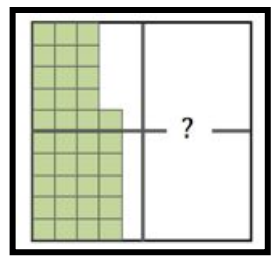
*The blocks are organized into 5 and 10 collections to encourage and practice collection thinking.*
*Five is not a guess, ten is not a guess.*
*There are 2 fives in ten, 5 tens is half the 100.*
*Rather than count, I can see quickly if the lengths do not match.*

**AREA Models : Number Relationships & Properties**
The Commutative property, the Associative Property, Inverse Operations

Students discuss and describe addition and subtraction relationships in the grids. The imagery is extended to encourage mentally comparing the quantities, then recording those manipulations to demonstrate understanding.

*I see 36 in the green.*
*Add 47 to the 36. Can you see the sum?*



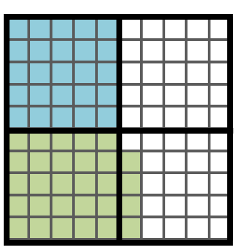




**100 -? =36**
**36 +? = 100**

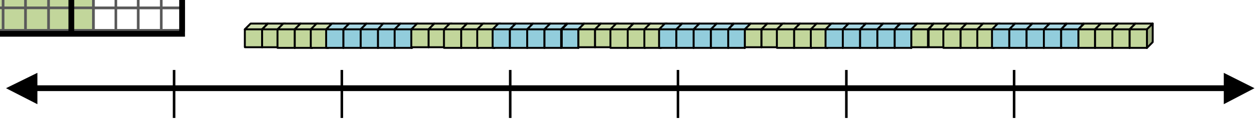
*Can you use the visual model to explain these equations?*

*The range of mental manipulative activities with the Hundred grid is explored in more depth in the BERCS for Place Value materials . The cards are Models to encourage thinking.*

Students are also encouraged to link the blocks to the 100 grid and to the numberline. They can literally cut the grid apart and stretch the number out to a numberline orientation.



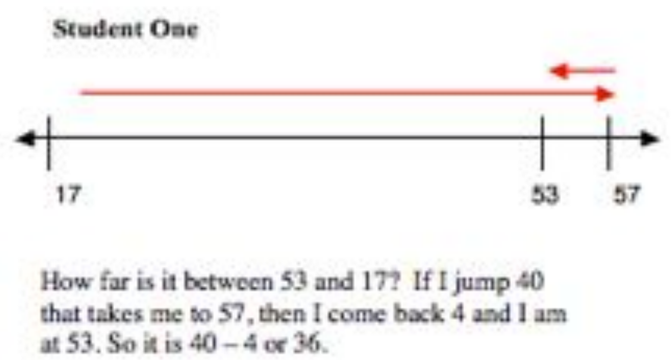
**TRANSLATE the AREA to a LINEAR MODEL**
**VISUALIZE Relationships**

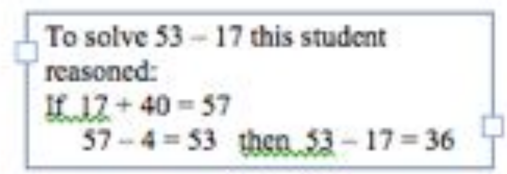


**The Goal:** *Students build and apply rich understandings of and a connected "sense" for numbers and number relationships that they can fluently and efficiently apply to mentally solve addition and subtraction equations with two digit numbers. The example below is the written explanation of what the student did MENTALLY. It is on paper only to demonstrate the thinking.*

Practice tasks include practicing mental strategies comparing, evaluating and analyzing solution strategies (Computational puzzles contextualized in word and in number puzzles)

**There were 53 candies hidden in the classroom. We found 17. How many are still hidden?**




| | |
|---|---|
| 14 | 23 |
| 17 | 6 |

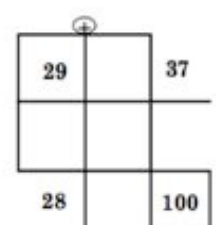
**?**

*Compare ways to solve mentally*

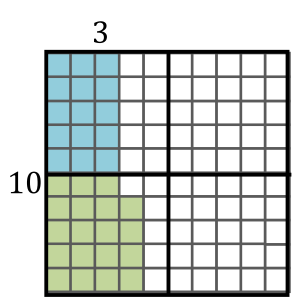
| | |
|---|---|
| 32 | 17 |
| 13 | ? |

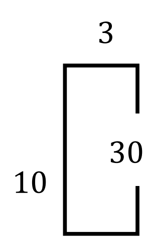
80

*Explain how you know what's missing?*




*GH Wheatley 2 Ways (Coming to Know Number, 1999/2010)*



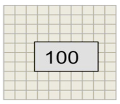




(3 x 10) + 4 = 34
34 ÷ 10 = 3 with a remainder of 4
4 of the next set of 10 or 4/10 which can be written 0.4 so 3.4 by Grade 6.

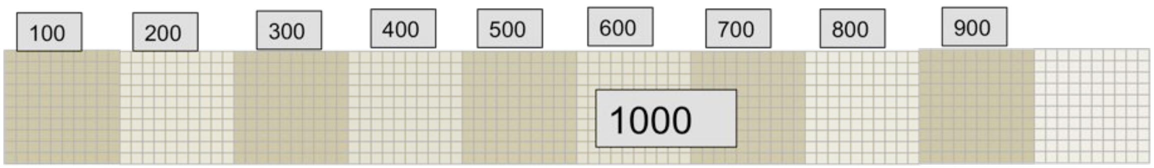
***Place Value is Multiplication***

# <u>**Area Models** for Reasoning about Whole Numbers BASE TEN Number System</u>

Place Value to the millions




*If this is the thousand describe what 3456 would look like in the model?*
*How about ten thousand?*




The Goal is not to draw pictures of the paper models. The goal is to have mental imagery to use to explain and describe numbers like this:

***600 000***
***240 000***
***390***
***9***


***844 399***


***842 399***

***800 000***
***42 000***
***390***
***9***

| | | | | | | |
|---|---|---|---|---|---|---|
| | | | | | | |
| *millions* | *hundred thousands* | *ten thousands* | *thousands* | *hundreds* | *tens* | *ones* |

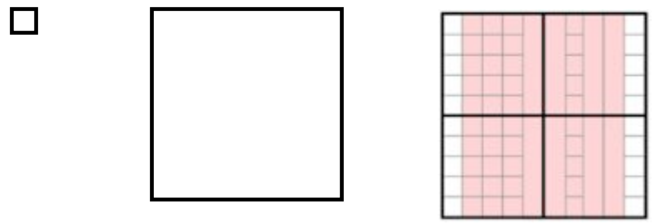
*Where does 24 ten thousands sit?*
*What about 39 tens?*

*Students can build and reason with the models for ten and one hundred to create numbers and explore flexible ways to record them up to a million using the area model. NOTE: Base ten "blocks" are actually a volume model, not an area model.*

## Area Models for Reasoning about Decimals in the BASE TEN Number System

When we build place value understandings with whole numbers, we start at the "one" and build in multiples of ten. 1, 10, 100, 1000
Decimals fractions as numbers are inside the whole numbers. We start by dividing the **ONE** into fractional parts.

***Start from the one, BLOW it up to see inside. Divide it into tenths, hundredths, thousandths***



***Describe the fractions as decimal fractions. Use words, use numbers. Position them on numberlines.***

*The pink card represents the decimal 0.8. Concretely I see 4 tenths and 40 hundredths but I could think 80 hundredths or 8 tenths.*

*The blue card represent 3.6 plus 0.06 which is one way to think of 3 and 66 hundredths.*
*36 tenths is an improper fraction. I can see it in the models.*

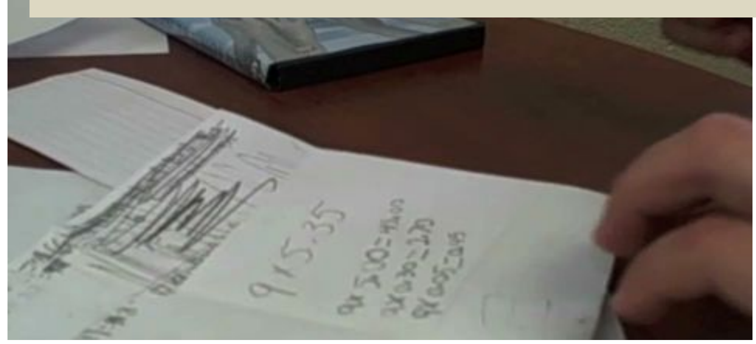


We will meet decimals in area models again once we have covered multiplication concepts.

**Mae Ling uses 3.12m to make a traditional dance outfit.**
**How much fabric will she use to make 8 outfits?** A grade 6 gave this oral explanation.
*I thought about 3 x 8 and then 0.12 x 8 so I added 24 and 0.96*

| | 3 | + 1/10 | + 2/100 |
|---|---|---|---|
| 8 | 24 | .8 | .16 |

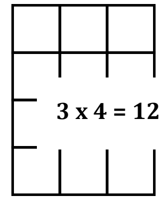
*This grade 6 student demonstrated 9 x 5.35 as*
*9 x 5 = 45.00 (He read 9 x five equals 45 wholes wholes)*
*9 x .3 = 2.7 (He read 9 x 3 tenths equals 27 tenths)*
*9 x .05 = 0. 45 (He read 9 x 5 hundredths equals 45 hundredths)*
*Then added in his head and gave the answer 4815 hundredths which is 48.15.*



***A variety of BERCS cards provide engaging practice tasks built around visualization and spatial reasoning to help students build rich and effective mental images for decimal numbers.***

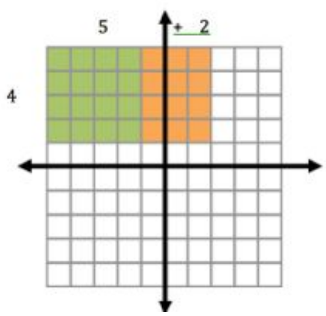
# Area Models for Reasoning about Multiplication

## Multiplication "FACTS" and Properties: Commutative Property, Distributive Property

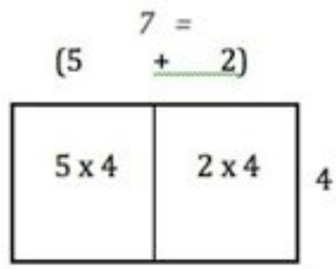



Students use tiles, Cuisenaire rods and/or grid paper to build multiplication arrays.
Very small arrays can be used as "flashcards" and used as building blocks to compose larger multiplication relationships. If students have a solid base in facts to 5 x 5, they can use them to create larger facts...
We do not want them counting every little square.

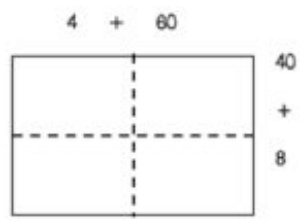
The physical arrays can then be represented with area models embedded in the hundred grid to reduce the need to count and in area diagrams

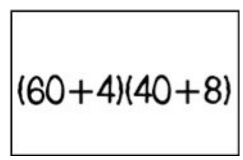



The arrays are embedded in the hundred grid to prompt number recogntion without counting

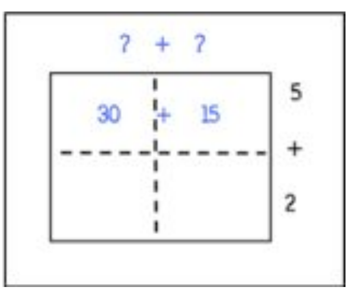


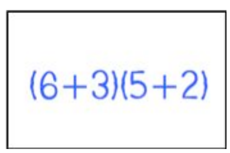

The representations do not show all the squares. Students are encouraged to think in arrays.



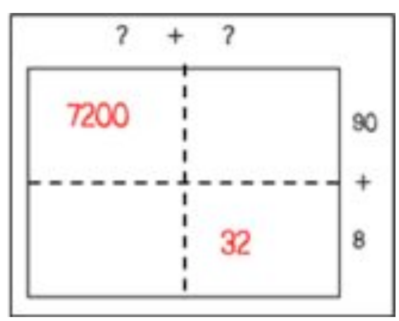

Do these match? Solve and explain.



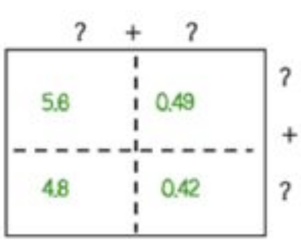




| | |
|---|---|
| 30 | 15 |
| 12 | 6 |

The distributive property is embedded in the imagery as students practice single and multi digit multiplications. Missing product cards can prompt division thinking as well.




Use multiplication and division equations in your explanation.

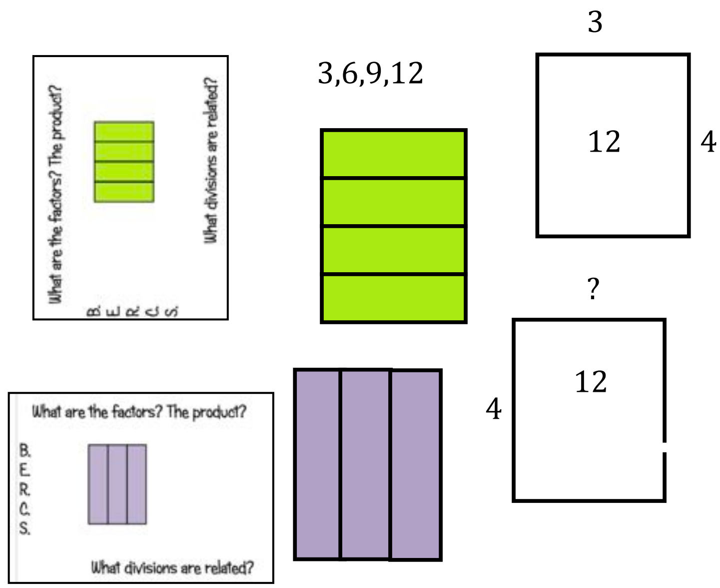


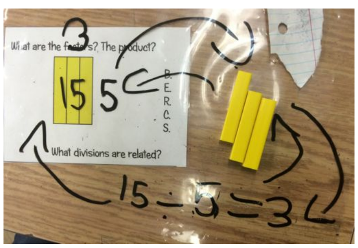


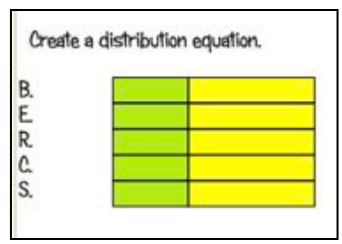
The model translates to decimals by Grade 6. The puzzle quality of the area representation keeps students actively thinking and practicing the distributive property.

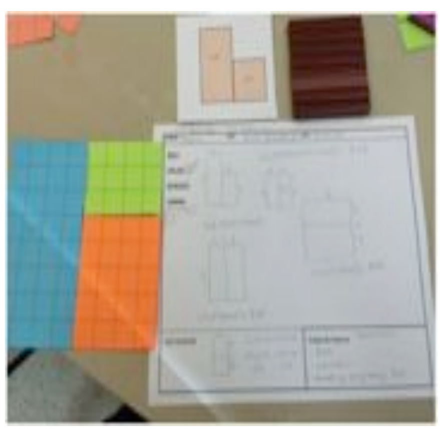
The Cuisenaire rods can be very useful as they demonstrate a growth as "units".
Teacher who have Cuisenaire rods use them instead of grid paper and tiles to help students build their initial "sense" of multiplication.
There are no little squares to interfere.
These cards are easily linked to the diagrams for area models that students draw .

You can build areas using the 3 rod and describe them counting by threes.



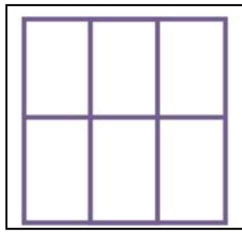



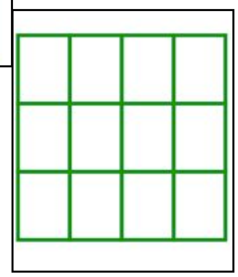
# Area Models for Reasoning about Fractions Multiplicatively

One Paper, 6 parts each part is one sixth we created it by dividing one paper into 3 parts. Three parts into 2 parts each? Or did we divide one paper into 2 parts, then each of those parts into 3 each? Either way we have 6 parts. The image reminds me of 3 and 2, hmmmm.

1÷ 2 = 1/3 1/3 ÷ 2 = 1/6

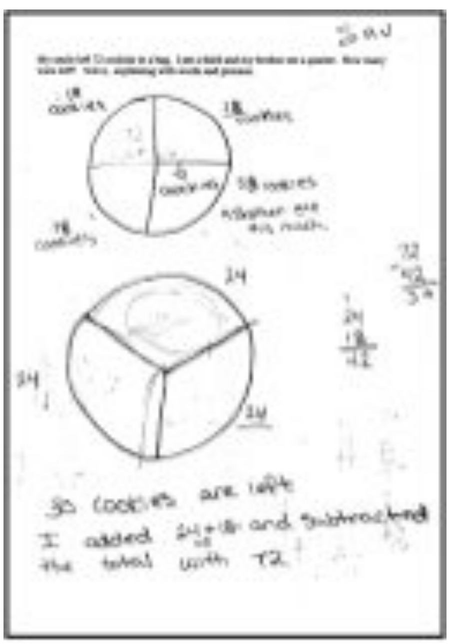
or is it 1 ÷ 6 = 1/6
the 1 has become 6/6

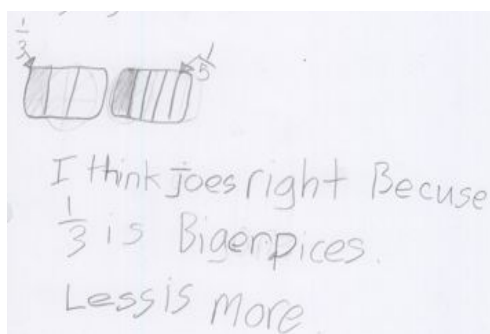




To create twelfths I can fold in fourths one way, then in thirds the other.

Or could I say 1 ÷ 4 = 1/4
Now 1/4 ÷ 3 = 1/12
HMMMMM




**The problem:**
Joe offered his brother one third of a chocolate bar or one fifth of the same chocolate bar. His brother said, "I want one fifth. That's the bigger piece. Joe said "no, one third is more than one fifth."

**The problem:**
There were 72 cherries. My brother ate one quarter and my sister ate one third. How many cookies are left for me?