Exploding Dots™

HANDOUTS

Experience 3:
Addition and Multiplication

Handout A: Addition ................................................................. 2
Solutions to Handout A ............................................................ 3
Handout B: Multiplication ......................................................... 4
Solutions to Handout B ............................................................ 5
Handout C: Wild Explorations .................................................... 6

© 2017 James Tanton. Some rights reserved. gdymath.com
This work is licensed under a
Creative Commons Attribution-NonCommercial-ShareAlike 3.0 Unported License (CC BY-NC-SA 3.0)

**Handout A: Addition**

Here is the *Exploding Dots* way to add 358 and 287.

\[
\begin{array}{c}
358 \\
+ 287 \\
\hline
= \\
\hline
\end{array}
\]

\[
\begin{array}{c}
\bullet \blacklozenge \blacksquare \\
\bullet \blacklozenge \blacksquare \\
\hline
\blacklozenge \blacklozenge \blacksquare \blacksquare \\
\end{array}
\]

5 | 13 | 15

Explosions then show that this answer is equivalent to 645.

Write down the answers to the following addition problems working left to right and not worrying about what society thinks! Then, do some explosions to translate each answer into something society understands.

\[
\begin{array}{ccc}
148 & 567 & 377 & 582 \\
+ 323 & + 271 & + 188 & + 714 \\
\hline
= & = & = & = \\
\hline
310462872 & 87263716381 & & \\
+ 389107123 & + 18778274824 & & \\
\hline
= & = & &
\end{array}
\]
Solutions to Handout A

148 + 323 = 4 | 6 | 11 = 471

567 + 271 = 7 | 13 | 8 = 838

377 + 188 = 4 | 15 | 15 = 5 | 5 | 15 = 565

582 + 714 = 12 | 9 | 6 = 1 | 2 | 9 | 6 = 1296

310462872 + 389107123 = 6 | 9 | 9 | 5 | 6 | 9 | 9 | 9 | 5 = 699569995

87263716381 + 18778274824 = 9 | 15 | 9 | 13 | 11 | 9 | 8 | 10 | 11 | 10 | 5
= ... = 106041991205
Handout B: Multiplication

We see that

\[ 26417 \times 3 = 6 \mid 18 \mid 12 \mid 3 \mid 21 \]

\[ \begin{array}{cccc} 2 & 6 & 4 & 1 \end{array} \begin{array}{c} 7 \end{array} \times 3 = \begin{array}{cccc} 6 & 18 & 12 & 3 \end{array} \begin{array}{c} 21 \end{array} \]

With explosions, this answer can be rewritten 79251.

Here are some more questions you might or might not choose to ponder.

Compute each of the following: 26417 \times 4, 26417 \times 5, and 26417 \times 9.

Compute 26417 \times 10 and explain why the answer has to be 264170.

(This answer looks like the original number with the digit zero tacked on to its end.)

Extra: Care to compute 26417 \times 11 and 26417 \times 12 too?

(The answer could be “No! I do not care to do this!”)
Solutions to Handout B

We have

\[26417 \times 4 = 8 | 24 | 16 | 4 | 28 = 10 | 4 | 16 | 4 | 28 = 105668\]
\[26417 \times 5 = 10 | 30 | 20 | 5 | 35 = 10 | 30 | 20 | 8 | 5 = 132085\]
\[26417 \times 9 = 18 | 54 | 36 | 9 | 63 = 18 | 54 | 36 | 15 | 3 = 237753\]

\[26417 \times 10 = 20 | 60 | 40 | 10 | 70 = \cdots = 264170\]

and

\[26417 \times 11 = 22 | 66 | 44 | 11 | 77 = \cdots = 290587\]
\[26417 \times 12 = 24 | 72 | 48 | 12 | 84 = \cdots = 317004\]

For a full discussion as to why \(26417 \times 10\) is 264170 have a look at Lesson 3.5 of [http://gdaymath.com/courses/exploding-dots/](http://gdaymath.com/courses/exploding-dots/).
**Exploding Dots**

**Experience 3: Addition and Multiplication**


**Handout C: WILD EXPLORATIONS**

Here are some “big question” investigations you might want to explore, or just think about. Have fun!

**EXPLORATION 1: THERE IS NOTHING SPECIAL ABOUT BASE TEN FOR ADDITION**

Here is an addition problem in a $1 \leftarrow 5$ machine. (That is, it is a problem in base five.) This is not a $1 \leftarrow 10$ machine addition.

\[
20413 + 13244 = 3|3|6|5|7 = 3|4|1|5|7 = 3|4|2|0|7 = 3|4|2|1|2 = 34212.
\]

<table>
<thead>
<tr>
<th>Here are the answers so that you can check your clever thinking.</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sum, as a $1 \leftarrow 5$ machine problem, is</td>
</tr>
</tbody>
</table>
| \[
20413 + 13244 = 3|3|6|5|7 = 3|4|1|5|7 = 3|4|2|0|7 = 3|4|2|1|2 = 34212. 
\] |
| In a $1 \leftarrow 5$ machine, 20413 is two 625’s, four 25’s, one 5, and three 1’s, and so is the number 1358 in base ten; 13244 is the number 1074 in base ten; and 34212 is the number 2432 in base ten. We have just worked out $1358 + 1074 = 2432$. |
**EXPLORATION 2: THERE IS NOTHING SPECIAL ABOUT BASE TEN FOR MULTIPLICATION**

Let’s work with a $1 \leftarrow 3$ machine.

a) Find $111 \times 3$ as a base three problem. Also, what are $1202 \times 3$ and $2002 \times 3$?

Can you explain what you notice?

Let’s now work with a $1 \leftarrow 4$ machine.

b) What is $133 \times 4$ as a base four problem? What is $2011 \times 4$? What is $22 \times 4$?

Can you explain what you notice?

*In general, if we are working with a $1 \leftarrow b$ machine, can you explain why multiplying a number in base $b$ by $b$ returns the original number with a zero tacked on to its right?*