GLOBAL MATH PROJECT

Uplifting Mathematics for All

Exploding Dots™ Teaching Guide

Experience 4:

Subtraction

Related resources:

- Access videos of *Exploding Dots™* lessons at:
  http://gdaymath.com/courses/exploding-dots/
- Be sure to review the *Getting Started* guide, available here.
- Printable student handouts for this experience are available here.

# Experience 4: Subtraction
Overview

Student Objectives

There is no such thing as subtraction! Students understand subtraction as the addition of the opposite. With the invention of antidots they can now perform subtraction in a 1 ← 10 machine via addition and explain the notion of "carrying" in the traditional algorithm.

The Experience in a Nutshell

There is no such thing as subtraction! Subtraction is just the addition of the opposite.

So let's invent the notion of the opposite of a dot – and antidot (drawn in white). A dot and an antidot together, like matter and antimatter, simply annihilate – POOF! – to leave nothing.
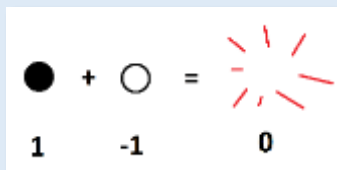
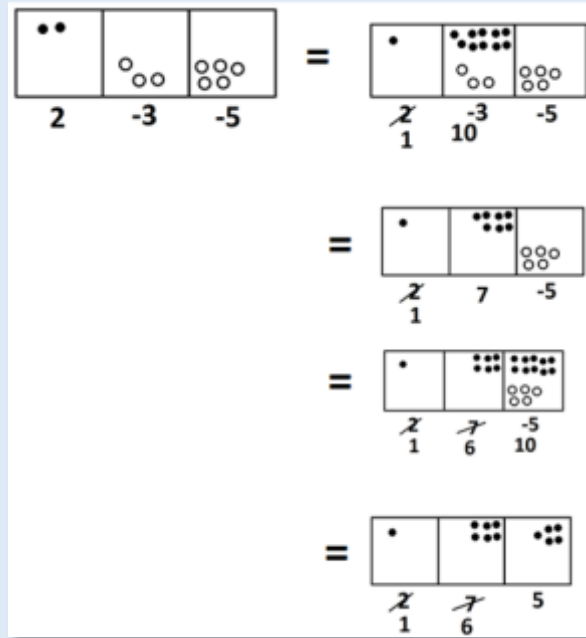We can now perform subtraction in a 1 ← 10 machine simply by adding antidots.

By "unexploding" dots, we can fix this (mathematically solid) answer to one that society prefers.

Of course, our original answer of $2 \mid -3 \mid -5$ can be interpreted as 200 and -30 and -5, also giving 165.

Setting the Scene

View the welcome video from James to set the scene for this experience:
http://gdaymath.com/lessons/explodingdots/4-1-welcome/ [0:45 minutes]

Piles and Holes; Dots and Antidots

This is Core Lesson # 9, corresponding to Lesson 4.2 on gdaymath.com/courses/exploding-dots/.

James has a video of this lesson here:

http://gdaymath.com/lessons/explodingdots/4-2-piles-holes-dots-antidots/ [2:35 minutes]

Here is the script James follows when he gives this lesson on a board. Of course, feel free to adapt this wording as suits you best. You will see in the video when and how James draws the diagrams and adds to them.

So far, we've made sense of addition and multiplication. But we skipped over subtraction. Why? Because I don't believe in subtraction! To me, subtraction is just the addition of the opposite.

At this point we have a choice. For younger students, I might give this story of piles and holes.

My disbelief in subtraction comes from another story that isn't true. Briefly, it goes as follows.

As a young child, I used to regularly play in a sandbox. And there I discovered the positive counting numbers as piles of sand: one pile, two piles, and so on. And I also discovered the addition of positive numbers simply by lining up piles. For example, I saw that two plus three equals five simply by lining up piles like this.
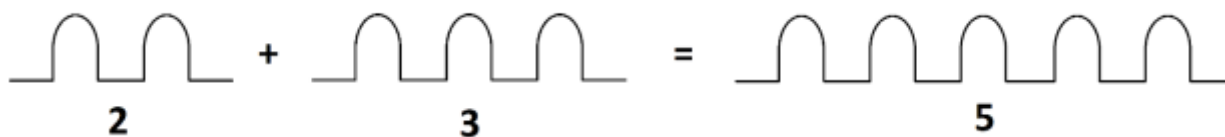

I had hours of fun counting and lining up piles to explore addition.

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But then one day I had an astounding flash of insight! Instead of making piles of sand, I realized I could also make holes. And I saw right away that *a hole is the opposite of a pile: place a pile and a hole together and they cancel each other out.* Whoa!
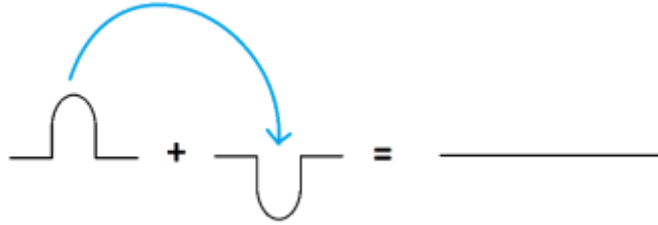
Later in school I was taught to call a hole "-1", and two holes "-2," and so on and was told to do this thing called "subtraction." But I never really believed in subtraction. My colleagues would read $5 - 2$, say, as, "five take away two," but I was thinking of five piles and the addition of two holes. A picture shows that the answer is three piles.
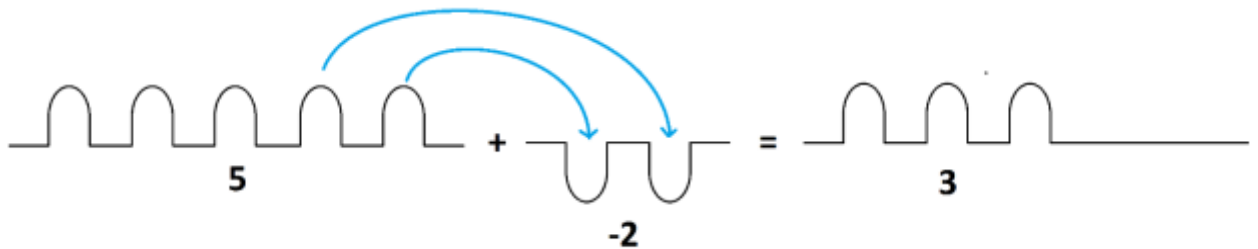
Yes. This gives the same answer as my peers, of course: the two holes "took away" two of the piles. But I had an advantage. For example, my colleagues would say that $7 - 10$ has no answer. I saw that it did.

$7 - 10 =$ seven piles and ten holes
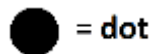
$=$ three holes

$= -3$

Easy!

Subtraction is just the addition of the opposite.

(By the way, I will happily write $7 - 10$ as "$7 + -10$." This makes the thinking more obvious.)

For an older audience I start here, and continue-on with younger students here too.

In our 1 ← 10 machine we've been working with dots, which I've been drawing as solid dots.

● = dot

We need the notion of the opposite of dot.

Now I've got two problems: *What do I call the opposite of a dot? And what do I draw for the opposite of a dot?*

I kid you not, every adult audience I have worked with so far has suggested the name "antidot" for the opposite of a dot, and every audience with just kids as suggested the name "tod," the word *dot* backwards. I go with whatever name the audience suggests. (In these notes I am using "antidot.")

Some students suggest I draw X's for antidots, and I suggest we don't, as X is so often used in mathematics and it might get confusing if we start doing algebra with our *Exploding Dots* machines. I steer students to drawing open dots for antidots.

Okay, let's draw an open circle for the opposite of a dot and call it an "antidot" (or "tod").
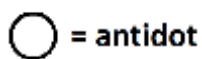
○ = antidot

Like matter and antimatter, which each annihilate one another when brought together, a dot and an antidot should also annihilate too – POOF! – when brought together to leave nothing behind.

● + ○ =

1   -1   0
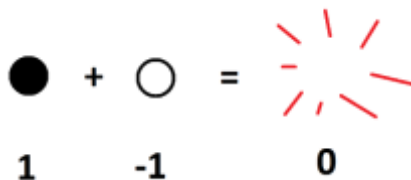
I usually stop here. But with younger audiences it might be good to add the following examples.

And we can conduct basic arithmetic with dots and antidots, just like we did with piles and holes.
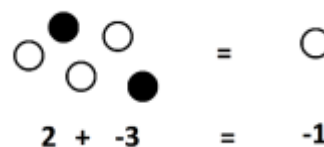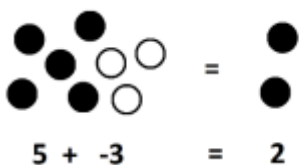
5 + -3   =   2

2 + -3   =   -1

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Subtraction

James has a video of this lesson here:

Okay. Onto subtraction.

Consider this subtraction problem.

$$536 - 123$$
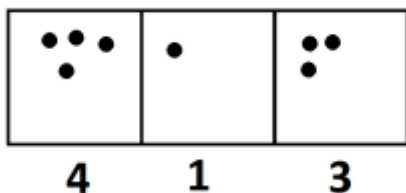
To me, this is 536 plus the opposite of 123.

The first number, 536, looks like this in a $1 \leftarrow 10$ machine: five dots, three dots, six dots.
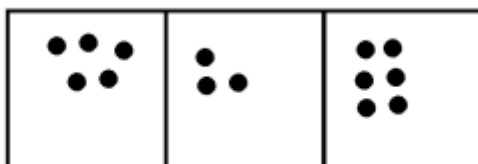
To this we are adding the opposite of 123. That is, we're adding one anti-hundred, two anti-tens, and three anti-ones.
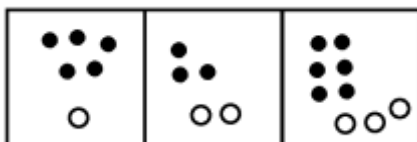
And now there are a lot of annihilations.: POOF!; POOF POOF!; POOF POOF POOF!

We see the answer 413 appear.

And notice, we get this answer as though we just work left to right and say

5 take away 1 is 4,

3 take away 2 is 1,

and

6 take away 3 is 3.

Yes! Left to right again!

$$
\begin{array}{r}
536 \\
-\ 123 \\
\hline
413
\end{array}
$$

All right. That example was too nice. How about 512 – 347?

$$
\begin{array}{r}
5\ 1\ 2 \\
-3\ 4\ 7 \\
\hline
\end{array}
$$

Going from left to right, we get: 5 take away 3 is 2, 1 take away 4 is –3, and 2 take away 7 is –5. The answer is two-hundred negative-three-ty negative-five.
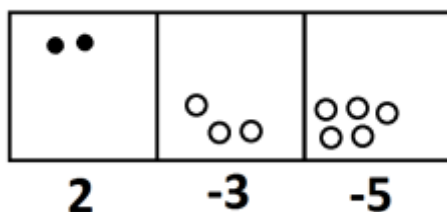
$$
\begin{array}{r}
5\ 1\ 2 \\
-3\ 4\ 7 \\
\hline
2|\text{-}3|\text{-}5
\end{array}
$$

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And this answer is absolutely, mathematically correct! The picture shows it is.

Here's five hundreds, one ten and two ones together with three anti-hundreds, four anti-tens, and seven anti-ones.
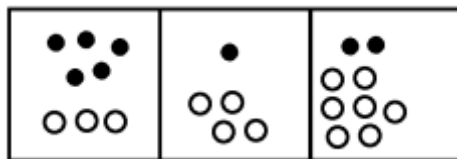
And after lots of annihilations we are left with two actual hundreds, three anti-tens, and five anti-ones.

The answer really is two-hundred negative-three-ty negative-five!

> But, of course, saying the answer to our subtraction problem this way seems mighty weird to society. *Can we fix up this mathematically correct answer for society's sake?*

This is a moment for students to ponder. Eventually someone has the flash of insight to suggest "unexploding" a dot.

Great! Let's unexplode dots! Any dot in a box to the left must have come from ten dots in the box just to its right, so we can just unexplode it to make ten dots.

Question: What sound effect should we make for unexploding?

The students and I usually settle for a sucking sound.

Okay. Let's unexplode one of the two dots we have in the leftmost box. Doing so gives this picture.

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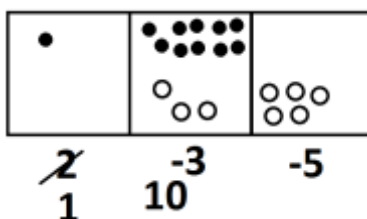
After annihilations, we see we now have the answer one-hundred seventy negative-five. Beautiful!
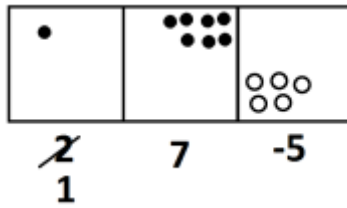
Let's unexplode again.

And with some more annihilations we see an answer society can understand: one-hundred sixty-five.

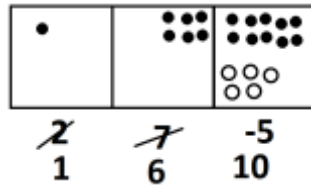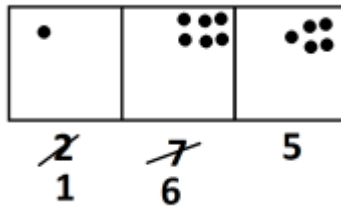## Optional Section: *The Traditional Algorithm*

This lesson is not one of the 15 core lessons; it is optional, corresponding to Lesson 4.4 on gdaymath.com/courses/exploding-dots/.

James has a video of this optional lesson here:

>   http://gdaymath.com/lessons/explodingdots/4-4-optional-traditional-algorithm/ [2:25]

How does this dots-and-boxes approach compare with the standard algorithm?

Consider again 512 – 347.

$$\begin{array}{r} 5\,1\,2 \\ -3\,4\,7 \\ \hline \end{array}$$

The standard algorithm has you start at the right to first look at "2 take away 7," which you can't do.

(Well you can do it, it is –5, but you are not to write that for this algorithm.)

So, what do you do?

You "borrow one." That is, you take a dot from the tens column and unexplode it to make ten ones. That leaves zero dots in the tens column. We should write ten ones to go with the two in the ones column.

$$\begin{array}{r} \overset{0}{5}\,\overset{10}{\cancel{1}}\,2 \\ -3\,4\,7 \\ \hline \end{array}$$

But we are a bit clever here and just write 12 rather than 10 + 2. (That is, we put a 1 in front of the 2 to make it look like twelve.)

$$\begin{array}{r} \overset{0}{5\,\cancel{1}\,^1 2} \\ -3\;4\;7 \\ \hline \end{array}$$

Then we say, "twelve take seven is five" and write that answer.

$$\begin{array}{r} \overset{0}{5\,\cancel{1}\,^1 2} \\ -3\;4\;7 \\ \hline 5 \end{array}$$

The rightmost column is complete. Shift now to the middle column.

We see "zero take away four," which can't be done. So, perform another unexplosion, that is, another "borrow," to see 10 – 4 in that column. We write the answer 6.

We then move to the last remaining column where we have 4 – 3, which is 1.

$$\begin{array}{r} \overset{4\,\overset{1}{0}}{\cancel{5}\,\cancel{1}\,^1 2} \\ -3\;4\;7 \\ \hline 6\;5 \end{array} \qquad \begin{array}{r} \overset{4\,\overset{1}{0}}{\cancel{5}\,\cancel{1}\,^1 2} \\ -3\;4\;7 \\ \hline 1\;6\;5 \end{array}$$

Phew!

Again. All correct approaches to mathematics are correct, and it is just a matter of style as to which approach you like best for subtraction. The traditional algorithm has you work from right to left and do all the unexplosions as you go along. The dots-and-boxes approach has you "just do it!" and conduct all the unexplosions at the end. Both methods are fine and correct.

Handout A: *Subtraction*

Use the student handout shown below for students who want practice questions from this lesson to mull on later at home. This is NOT homework; it is entirely optional. (See the document "Experience 4: Handouts" for a printable version.)

Exploding Dots

Experience 4: Subtraction

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout A: *Subtraction*

We can now perform subtraction in a $1 \leftarrow 10$ machine simply by adding antidots. (Some people prefer to call them tods.)

Unexplosions then show that this answer is the same as 165.

Here is a question for you to try, if you like.

Compute each of the following two ways: the dots-and-boxes way (and fixing the answer for society to read) and then with the traditional algorithm. The answers should be the same.

$$6328 - 4469$$

$$78390231 - 32495846$$

Thinking question along the way: As you fix up your answers for society, does it seem easier to unexplode from left to right, or from right to left?

Additional question: Do you think you could become just as speedy the dots-and-boxes way as you currently are with the traditional approach?

Solutions to Handout A

$$6328 - 4469 = 2\,|\,{-1}\,|\,{-4}\,|\,{-1} = 1\,|\,9\,|\,{-4}\,|\,{-1} = 1\,|\,8\,|\,6\,|\,{-1} = 1\,|\,8\,|\,5\,|\,9 = 1859$$

$$
\begin{aligned}
78390231 - 32495846 &= 4\,|\,6\,|\,{-1}\,|\,0\,|\,{-5}\,|\,{-6}\,|\,{-1}\,|\,{-5} \\
&= 4\,|\,5\,|\,9\,|\,0\,|\,{-5}\,|\,{-6}\,|\,{-1}\,|\,{-5} \\
&= 4\,|\,5\,|\,8\,|\,10\,|\,{-5}\,|\,{-6}\,|\,{-1}\,|\,{-5} \\
&= 4\,|\,5\,|\,8\,|\,9\,|\,5\,|\,{-6}\,|\,{-1}\,|\,{-5} \\
&= 4\,|\,5\,|\,8\,|\,9\,|\,4\,|\,4\,|\,{-1}\,|\,{-5} \\
&= 4\,|\,5\,|\,8\,|\,9\,|\,4\,|\,3\,|\,9\,|\,{-5} \\
&= 4\,|\,5\,|\,8\,|\,9\,|\,4\,|\,3\,|\,8\,|\,5 \qquad = 45894385
\end{aligned}
$$

I personally find it much easier to do the unexplosions from left to right.

Handout B: *Wild Explorations*

Use the student handout shown below for students who want some deep-thinking questions from this Experience to mull on later at home. This is NOT homework; it is entirely optional, but this could be a source for student projects. (See the document "Experience 4: Handouts" for a printable version.)

Exploding Dots

Experience 4: Subtraction

Access videos of all *Exploding Dots* lessons at: http://gdaymath.com/courses/exploding-dots/

Handout B: *WILD EXPLORATIONS*

Here are some "big question" investigations you might want to explore, or just think about. Have fun!

EXPLORATION 1: IS THERE ANOTHER WAY TO INTERPRET THE DOTS-AND-BOXES ANSWERS?

When Sunil saw,

$$
\begin{array}{r}
5\ 1\ 2 \\
-3\ 4\ 7 \\
\hline
2\,|\text{-}3\,|\text{-}5
\end{array}
$$

he wrote on his paper the following lines:

$$
\begin{array}{r}
200 \\
-30 \\
-5
\end{array}
$$

He then said that the answer has to be 165.

    a) Can you explain what he is seeing and thinking?

    b) What would Sunil likely write on the page for 7109 – 3384?

EXPLORATION 2: WHAT ABOUT NEGATIVE ANSWERS?

How might you handle and interpret this subtraction problem?

$$
\begin{array}{r}
148 \\
-\ 677
\end{array}
$$

15