



# Introduction

Funville Adventures is a math-inspired fantasy that introduces children to the concept of functions, which are personified as magical beings with powers. Each power corresponds to a transformation such as doubling in size, rotating, copying, or changing color. Some Funvillians have siblings with opposite powers that can reverse the effects and return an object to its original state, but other powers cannot be reversed. In this way, kids are introduced to the mathematical concepts of invertible and non-invertible functions, domains, ranges, and even functionals, all without mathematical terminology.

This story is supplemental to the book, and features the Funvillian characters of Harvey, Doug, Cory, Marge, Constance, and Connie. Harvey and Doug are siblings whose powers are complimentary: Harvey can make an object scale down to half its original size, while Doug can make an object scale up to twice its original size. Cory and Marge are also siblings: Cory can copy an object, producing two identical objects, and Marge can merge two identical objects back into one. Constance's power is to turn any object into an elephant. Connie's power allows her to instantly count the number of any particular kind of object.

[Exploding Dots](#) is the name of an astounding mathematical story that starts at the very beginning of mathematics – it assumes nothing – and swiftly takes you a wondrous journey through grade school arithmetic, high-school polynomials algebra, infinite sums, and advanced mathematics and unsolved research problems baffling mathematicians still to this day.

Explore both stories for a rich crossover experience.



## Chair Marks the Spot

Once a year, a barbershop quartet who call themselves the “Funville Four” perform a concert for the other Funvillians. Liza sings the soprano part, Heather sings Alto, Connie keeps the rhythm as a bass part, and Randy sings a random harmony. It was the day of the concert and the four of them had just finished carefully arranging all of the chairs in the concert hall when Harvey, Doug, Cory, Marge, and Constance entered.

“The concert doesn’t start for another couple of hours,” Connie told the newcomers. “In the meantime, we are going off to rehearse somewhere else so we won’t be overheard. You’re welcome to stay here, but make sure you don’t disturb the chairs. If any of them are missing when we get back, I *will* know.”

“Don’t worry,” Doug promised. “When you get back, the chairs will be just as you left them.”

With that promise secured, the Funville Four waved goodbye and went off to rehearse. Cory surveyed the concert hall, taking in the orderly arrangement of chairs covering the room. The pattern on the carpet was divided into black and white squares, forming a grid. Each chair was centered carefully on its own white square of carpet, facing the stage.

“I want to play a game,” Corey said. “But we’d need more space.”

“What do you want to play?” Marge asked.

“Cory Says,” Cory said. “It’s like Simon Says, but where I am Simon.”

“I’ll play,” Harvey volunteered.

“Me too,” said Doug.

“I’ll play too if Constance joins,” Marge said.

Constance shrugged.



“I’ll take that as a yes!” Cory declared. “Now we just need to move some of these chairs out of the way.”

“I have an idea!” Marge exclaimed. “These chairs are all identical. I could start merging them together to create space. After the game, Cory, you could copy them again so we can put them back.”

Cory nodded. “Good idea, Marge! That will give us plenty of space.”

But just as Marge was about to merge two chairs, Constance grabbed her arm. “Wait!” Constance implored. “How will we remember how many chairs there were?”

“Does anyone have a pencil and paper?” Marge asked. “We could count the chairs and then write down the number.”

Marge knew that she could not depend on them remembering the number because when situations like this arose in the past, different Funvillians would remember different amounts and there would be no end to the arguments.

“I had a pencil,” said Constance, reaching into her pocket. “But I already turned it into an elephant.” She held up the small gray elephant for the others to see.

“Why did you do that?” Marge asked.

Constance shrugged. “I like elephants more than pencils,” she said.

“Plus,” Constance added, “it would not be enough to just write down the number of chairs. We’d need to remember the arrangement as well.”

“That’s easy enough,” countered Cory. “We’d just need to arrange them in a square directly in front of the stage and make sure to put the chairs only on the white tiles.”

Everyone agreed that this should be easy enough to do once you had the correct number of chairs. But they still had the problem of not having anything to write with. For a moment, everyone was stumped.



“Maybe we don’t need to merge the chairs,” Harvey said. “I could shrink them down to be tiny and then we could fit them all in a very small space!”

Doug shook his head. “We might easily lose a tiny chair,” he worried. “And if we don’t remember how many there are, we won’t know how many we lost!”

Again they were stumped.

Doug furrowed his brow in concentration. “How about this,” he started. “What if as soon as Marge merges a chair, I double the size of the result? Then it will be *twice* as big, so we will know it represents *two* chairs.”

“Let’s try it!” Marge agreed. She merge two chairs, and Doug doubled the result.

“If we keep going,” Harvey reasoned aloud, “we would eventually get one GIANT chair.” He gestured widely with his arms to illustrate. “And then we could reverse the process. I would shrink the giant chair just once, and then Cory would copy it. And we could keep shrinking and copying until all the chairs were back to normal size.”

“How would we remember exactly what size was normal?” Marge asked.

“Easy!” Doug answered. “We’d keep one normal chair separate for comparison.”

Marge nodded. But Constance frowned. “I suspect there is a flaw in this plan,” she said quietly, but it was lost in the others’ excitement.

Marge continued merging chairs, and Doug dutifully doubled each result. But then Marge stepped back, surveying the ever larger chairs.

“Hmm....” she said. “I think there’s a problem.”

“What’s the problem?” Doug asked.

“The height of the chairs,” Marge said. “It’s growing very quickly.”

“But there are fewer chairs,” Doug countered. “So that’s progress!”



Marge shook her head. “Now that I think about it, one chair that is twice as big is *as tall* as two original chairs, one on top of the other. There may be fewer chairs in *number*, but soon each one will be taller than a tower of ten original chairs, and hence taller than the ceiling!”

Doug opened his mouth to argue, but stopped. He looked up at the largest chairs, which were in fact getting dangerously close to the ceiling. “I see your point,” he conceded.

“I tried to tell you,” Constance said.

“I’ll just shrink them!” Harvey volunteered. “That will be instant progress!”

“NO!” the others yelled in unison.

“You can’t,” Cory explained. “We’d lose track!”

“Oh right,” Harvey said. “Sorry guys, just a reflex.”

Just to be safe, they returned the chairs back to their original number and size.

“Is there some way to “mark” a chair as representing more chairs *without* making it big?” Doug asked.

They all thought very hard about this. “Too bad we don’t have a pencil,” Cory lamented, looking at Constance.

Constance shrugged and stared down at the tile floor. It was then that she had an idea.

“What if we use the *position* of a chair?” she asked.

“What do you mean?” Cory asked.

But Constance was too busy to explain. She had already started dragging chairs around, moving each onto its own square along the left wall.

“What are you doing?” Harvey asked.



“I’ve solved it!” Constance insisted. “I’ll show you. We start by putting one chair on each square along the left wall.”

The others dutifully helped to place chairs on the squares along the left wall of the room. When they ran out of spaces along the wall, Constance instructed Marge to bring the next chair to the first spot in the next column of squares.

“Now merge these two!” Constance insisted, pointing to the pair of adjacent chairs.

“Are you sure?” Marge asked, hesitating. “Are you certain we won’t lose track?”

“Trust me,” said Constance.

Marge obliged and merged the two chairs in the first row. Constance straightened the resulting single chair carefully, placing it in the second square of the first row, leaving the square along the left wall empty.

“You see?” Constance asked the others. “A chair in the first column of squares along the wall represents 1 chair. A chair in the second column of squares represents 2 chairs!”

Doug contemplated this for a moment. “I do see!” he declared. “That’s so neat!”

The others were also excited. They lined up more chairs in the second column of squares, and Marge merged the first column into them.

There were still a few more chairs that had not been dealt with. Doug began placing them in the first column.

“What now?” Cory asked.

Doug was quick with an answer. “We can merge two chairs from this 2nd column of squares and put the result in the third column”, he suggested.

“So if a chair is in the first column of squares, it represents 1 chair,” Harvey reasoned. “If it’s in the second column of squares, it represents 2 chairs. And if it is in the third column of chairs it represents...”



“Four chairs!” Marge shouted, gleefully merging two of the 2nd column chairs and placing the result in the third column.

She continued down the second column of squares, moving merged chairs into the third column and clearing space. Now chairs in the first column of squares could be merged and moved into the vacant spaces in the second column.

“I see the pattern too!” Harvey exclaimed. “If we merge two chairs in the 3rd column of squares and put the result in the 4th column, it represents  $2 \times 2 \times 2 = 8$  chairs!”

“That’s right,” said Constance approvingly. She watched as the chairs were steadily merged towards the right of the room, taking up less and less space.

In the end, there was one chair in each of the second, fifth, and sixth columns of squares. And finally there was plenty of space to play a game of “Cory says”!

After a few games, the Funville Four returned from their rehearsal.

“Oh no!” Connie said as she entered the concert hall and saw the three lonely chairs. “I told you not to change the chairs! Where did they all go?”

“Don’t worry!” Cory assured her. “We came up with a clever way to keep track of them so that we can put them all back!”

He quickly copied the chair in the second column to make 2 chairs and copied the chair in fifth column repeatedly to make 16 chairs. Then he copied the chair in the sixth column of squares repeatedly to make 32 chairs.

Together, they placed all of chairs back into a square arrangement, remembering to use only the white tiles.

“Weren’t we clever?” Cory bragged. “We came up with a cool way of keep track of how many chairs there were! That’s how we were able to put all  $2+16+32 = 50$  chairs back!”

“You know, you could have just noticed that there were 5 chairs per rows.” Connie pointed out. “Surely 5 is a small enough number even for you to remember.”





Corey, Marge, Harvey, and Doug all dropped their jaws in unison.

“That’s so obvious!” Doug said, slapping his hand on his forehead. “Why didn’t I think of that?”

Constance shrugged. “I like my way better,” she said.

