QUADRATICS
6.3 Practice

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SETTING THE SCENE

We now have two ways to work with quadratic equations and their graphs.

1. If a quadratic graph has vertex $x = 2$, $y = 3$ say, then we know the equation is of the form

$$y = a(x-2)^2 + 3$$

for some steepness factor a.

2. Identifying two interesting x values that represent symmetric points in an equation of the form

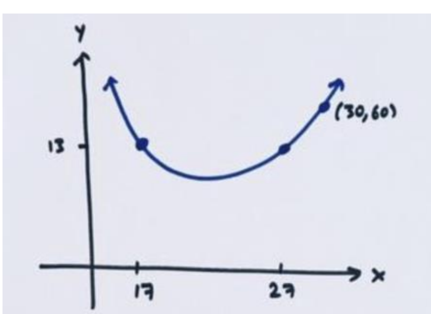
$$y = a(x-4)(x-12)+7$$

say, allows us to identify the line of symmetry of the graph and readily sketch its graph.

Let's practice these ideas some more.

PRACTICE

PROBLEM: *Find a quadratic equation whose graph is as shown.*



Answer: The graph shows two interesting symmetrical points and suggests then an equation of the form

$$y = a(x-17)(x-27)+13$$

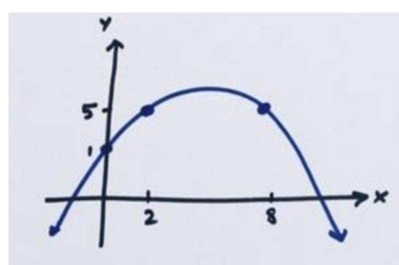
for some steepness a. Since the graph is also meant to pass through the point $x = 30$, $y = 60$ we must have that

$$60 = a(13)(3)+13$$

is a true number sentence. This forces $a = \frac{47}{39}$, a positive steepness larger than 1, as expected. The equation we seek is

$$y = \frac{47}{39}(x-17)(x-27)+13.$$

PRACTICE 1: *Find the quadratic equation whose graph appears as shown.*



PRACTICE 2: *Write down a quadratic equation whose graph has x intercepts $x = -3$ and $x = 11$ and y intercept 10.*

PRACTICE 3: *Sketch the graph of* $y = -3x^2 - 18x + 5$ *and then use the graph to rewrite the equation in "vertex form."*

Here's a typical textbook question.

PROBLEM: *Consider the equation* $y = 2x^2 - 8x + 6$.

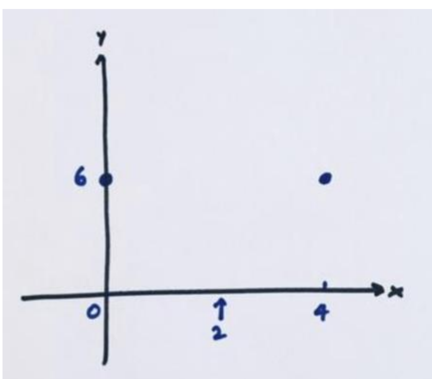
a) *Find its vertex.*
b) *Find its axis of symmetry.*
c) *Rewrite the equation in vertex form.*
d) *Sketch its graph.*
e) *Find the* x *intercepts of the graph.*
f) *Find the* y *intercept of the graph.*

Answer: Firstly, no one says that you must answer questions in the order presented to you. Since a picture tends to reveal all, let's just go straight to part d) and sketch a graph of the quadratic.

We have

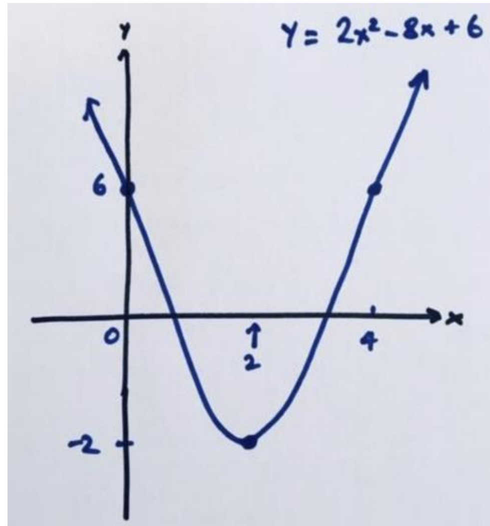
$$y = 2x(x-4) + 6$$

giving us $x = 0$ and $x = 4$ as interesting.



The line of symmetry is at $x = 2$, answering part b). (Some people call this the *axis of symmetry*.)

The vertex is on this line too. When $x = 2$ we have $y = 2(2)(-2) + 6 = -2$. So the vertex is $(2, -2)$, answering part a). And we have a good sketch of the graph for part d).



We also see that the answer to part f) is clear. The y intercept is the point $(0, 6)$.

To answer a), we can see the equation of this graph has the form $y = a(x-2)^2 - 2$. Since the original form of the equation contains "$2x^2$" it must be that $x = 2$. So the vertex form of the equation is

$$y = 2(x-2)^2 - 2.$$

This leaves now only part e).

An x intercept is a point with y-coordinate zero. So we are seeking the x-values which satisfy the equation

$$0 = 2x^2 - 8x + 6.$$

This is back to the algebra of quadratic equations. We can solve this any number of ways—with the square method, with the quadratic formula, perhaps with unsymmetrical factoring. We could also choose to work with an alternative form of the equation, say, with the vertex form of the equation from part a).

For fun, let's do that here. Let's solve

$$0 = 2(x-2)^2 - 2.$$

This gives

$$2(x-2)^2 = 2$$
$$(x-2)^2 = 1$$
$$x-2 = 1 \text{ or } -1$$
$$x = 3 \text{ or } 1.$$

The x intercepts are $(1,0)$ and $(3,0)$.

The point of the previous question was to show that YOU are in control of your own fabulous thinking and doing. Just use your wits and common sense and all will start to fall into place.

PRACTICE 4: *Consider the equation* $y = 3x^2 - 6x + 20$.

- *a) Find its vertex.*
- *b) Find its axis of symmetry.*
- *c) Rewrite the equation in vertex form.*
- *d) Sketch its graph.*
- *e) Find the* x *intercepts of the graph.*
- *f) Find the* y *intercept of the graph.*

PRACTICE 5: *Consider the equation* $y = 5x^2 - 10x$.

- *a) Find its vertex.*
- *b) Find its axis of symmetry.*
- *c) Rewrite the equation in vertex form.*
- *d) Sketch its graph.*
- *e) Find the* x *intercepts of the graph.*
- *f) Find the* y *intercept of the graph.*

PRACTICE 6: *Solve the following quadratic equations.*

- *a)* $(x-3)(x+5) = 1$
- *b)* $x^2 = (2x-1)(2x+1) - 5$
- *c)* $(x-10)(x+1) + 5 = 12$

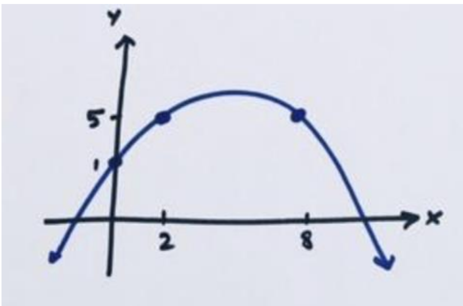
PRACTICE 7: *Find, in terms of* c, *the value* k *so that*

$$y = (x+c)(x-c) + k$$

gives -2 *as the smallest possible* y *value.*

SOLUTIONS

PRACTICE 1: *Find the quadratic equation whose graph appears as shown.*



Answer: The picture suggests an equation of the form $y = a(x-2)(x-8)+5$. When $x=0$ we should have $y=1$, so we need

$$1 = a(-2)(-8)+5$$

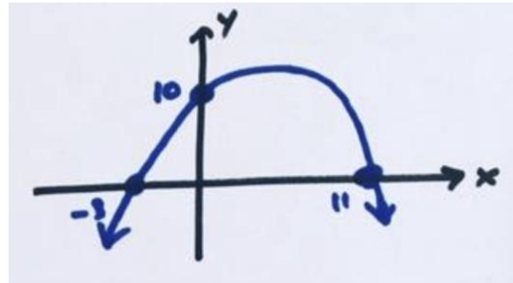
giving $a = -\frac{1}{4}$. So the equation is

$$y = -\frac{1}{4}(x-2)(x-8)+5.$$

PRACTICE 2: *Write down a quadratic equation whose graph has* x *intercepts* $x=-3$ *and* $x=11$ *and* y *intercept* 10.

Answer: The sketch suggests the equation

$$y = a(x+3)(x-11)+0.$$

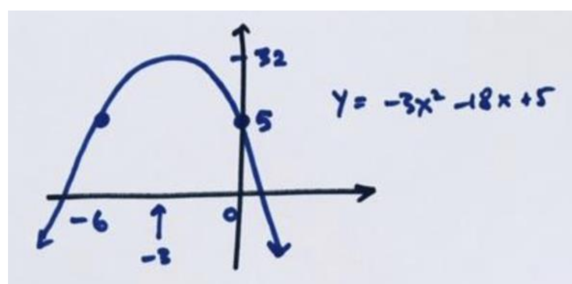


When $x=0$ we need $y=10$ giving $a = -\frac{10}{33}$. The equation is

$$y = -\frac{10}{33}(x+3)(x-11).$$

PRACTICE 3: *Sketch the graph of* $y=-3x^2-18x+5$ *and then use the graph to rewrite the equation in "vertex form."*

Answer: We have $y=-3x(x+6)+5$ showing the line of symmetry is at $x=-3$. For this x value, we have $y=-3(-3)(3)+5=32$. The graph thus appears



In vertex form, we must have

$$y = a(x+3)^2+32$$

with $a=-3$ to yield a "$-3x^2$" term when expanded. So the vertex form of the equation is

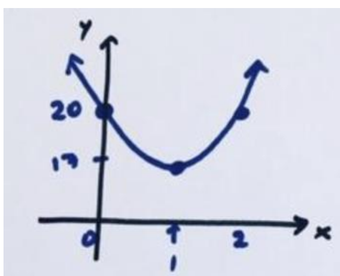
$$y=-3(x+3)^2+32.$$

PRACTICE 4: *Consider the equation* $y = 3x^2 - 6x + 20$.

a) Find its vertex.
b) Find its axis of symmetry.
c) Rewrite the equation in vertex form.
d) Sketch its graph.
e) Find the x intercepts of the graph.
f) Find the y intercept of the graph.

Brief Answer:

a) $(1,17)$
b) At $x = 1$
c) $y = 3(x-1)^2 + 17$
d)



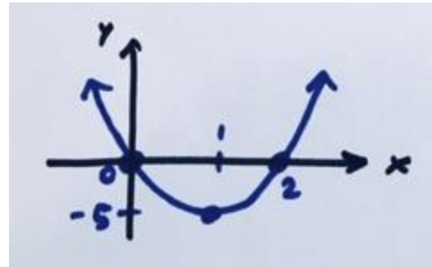
e) There are none
f) $(0,20)$.

PRACTICE 5: *Consider the equation* $y = 5x^2 - 10x$.

a) Find its vertex.
b) Find its axis of symmetry.
c) Rewrite the equation in vertex form.
d) Sketch its graph.
e) Find the x intercepts of the graph.
f) Find the y intercept of the graph.

Brief Answer:

a) $(1,-5)$
b) At $x = 1$
c) $y- = 5(x-1)^2 - 5$
d)



e) $(0,0)$ and $(2,0)$
f) $(0,0)$

PRACTICE 6: *Solve the following quadratic equations.*

a) $(x-3)(x+5) = 1$
b) $x^2 = (2x-1)(2x+1) - 5$
c) $(x-10)(x+1) + 5 = 12$

Brief Answers:

a) Rewrite as $x^2 + 2x - 15 = 1$. Solving gives $x = -1 + \sqrt{17}$ or $x = -1 = \sqrt{17}$.
b) Rewrite as $x^2 = 4x^2 - 1 - 5$, that is, as $x^2 = 2$ with solutions $x = \sqrt{2}$ or $x = -\sqrt{2}$.
c) Rewrite as $x^2 - 9x - 5 = 12$ which has solutions $x = \dfrac{9 \pm \sqrt{13}}{2}$.

PRACTICE 7: *Find, in terms of c, the value k so that*

$$y = (x+c)(x-c) + k$$

gives -2 as the smallest possible y value.

Brief Answer: We have that $x = c$ and $x = -c$ give symmetrical points on a symmetrical graph. The line of symmetry is thus at $x = 0$, and this is where the vertex lies.

At $x = 0$, $y = k - c^2$. We want this to equal -2 and so we must have

$$k = c^2 - 2.$$