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# QUADRATICS
# 6.4 More Practice

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## MORE PRACTICE

Here are a few more practice problems. This next one illustrates the interplay between geometry and algebra.

**PRACTICE 1:**

*a) Solve* $x^2 + 10x + 30 = 0$ *and see what happens when you try.*

*b) Sketch the graph of* $y = x^2 + 10x + 30$.

*c) Use the graph to explain what happened in part a)*

*d) Will* $x^2 + 10x + 30 = 11$ *have a solution? If so, how many solutions?*

*e) For which value(s)* $k$ *does* $x^2 + 10x + 30 = k$ *have only one solution?*

**PRACTICE 2:** *How many solutions does* $-x^2 + 4x - 5 = 0$ *have? Answer this question not by algebra, but by graphing.*

**PRACTICE 3:**

a) *Find a value* $k$ *so that the graph of*

$$y = 5x^2 - 10x + k$$

*just touches the* $x$ *axis.*

*b) Find a value* $m$ *so that* $y = -2x^2 - 18x + m$ *gives the highest output value of* $100$.

*c) Find a value* $p$ *so that* $y = (x-p)(x-3p)$ *has smallest output value of* $-10$.

Some more questions!

**PRACTICE 4:** *Find, in terms of* $r$, *a value* $k$ *so that the graph of* $y = 3x(2r - x) - k$ *just touches the* $x$ *axis.*

**PRACTICE 5:** *Consider the equation*

$$y = 2(x-1)(x+1) + (x-3)(x+4) + x(x-2)$$

*a) Is this an equation in the form* $y = ax^2 + bx + c$ *in disguise for some numbers* $a$, $b$, *and* $c$? *(This is just a YES/NO question!)*

*b) Explain how one readily sees* $a = 2+1+1 = 4$.

*c) Explain why putting* $x = 0$ *into the right side of the equation allows us to conclude that* $c = -14$.

*d) What is the value of* $b$?

**PRACTICE 6:** *A rectangle has side lengths* $7 - r$ *and* $3 + r$ *for some value* $r$. *What value for* $r$ *gives a rectangle of maximal area?*

**PRACTICE 7:** *Here are three quadratic equations:*

(A) $y = 3(x-3)(x+5)$

(B) $y = 2x^2 + 6x + 8$

(C) $y = 2(x-4)^2 + 7$

*i) For which of these three expressions is it very easy to answer the question: "What is the smallest $y$-value the expression can produce?"*

*ii) For which of these three expressions is it very easy to answer the question: "Where does the graph of the equation cross the $x$-axis?"*

*iii) For which of these three expressions is it very easy to answer the question: "Where does the graph of the quadratic cross the $y$ axis?"*

*iv) For which of these three expressions is it very easy to answer the question: "What are the coordinates of the vertex in this equation's graph?"*

**PRACTICE 8:** Here is a silly question.
*I make and sell BIPS. It costs me $80,000 + 200n$ rupees to make $n$ BIPS.*
*If I sell $n$ BIPS I bring in $2n(600-n)$ rupees.*

*a) Ignoring costs, what number of BIPS sold brings in the largest number of rupees?*

*b) What number of BIPS should I sell to make the biggest profit?*

## FINAL COMMENT

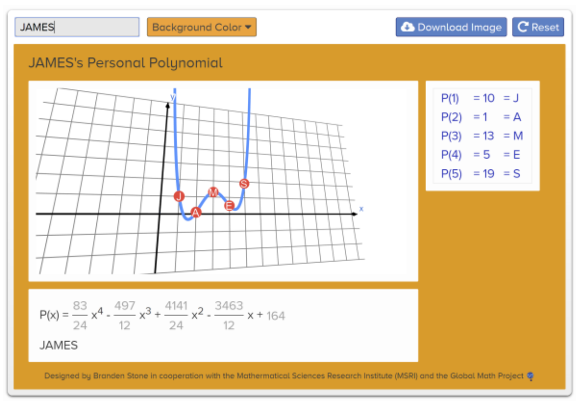
Do you recall at the start of this lesson me giving quadratic equations (and more complicated equations) that "spell" different versions of my name?

If you would like to learn all about that—and play with an automated program that will let you type in your own name and see the graph of your name!—go to

[www.globalmathproject.org/personal-polynomial](http://www.globalmathproject.org/personal-polynomial)

You'll find videos of me there explaining all the mathematics too.

ENJOY!



## SOLUTIONS

**PRACTICE 1:**

*a) Solve* $x^2 + 10x + 30 = 0$ *and see what happens when you try.*

*b) Sketch the graph of* $y = x^2 + 10x + 30$.

*c) Use the graph to explain what happened in part a)*

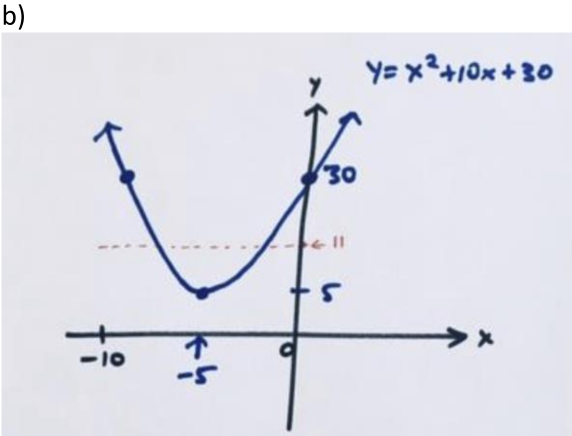
*d) Will* $x^2 + 10x + 30 = 11$ *have a solution? If so, how many solutions?*

*e) For which value(s)* $k$ *does* $x^2 + 10x + 30 = k$ *have only one solution.*

**Brief Answers:**

a) We have $(x+5)^2 = -5$. There are no solutions.

b)



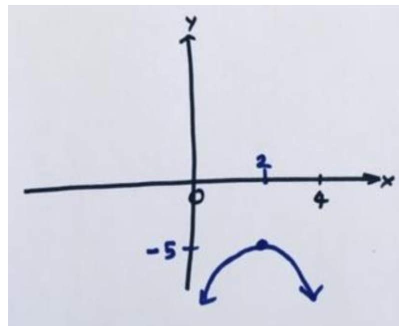

c) We see from the graph that there are no $x$ values that give a $y$ value of zero.

d) We see from the graph the there are two $x$ values that give $y = 11$.

e) We see from the graph that the answer is $k = 5$.

**PRACTICE 2:** *How many solutions does* $-x^2 + 4x - 5 = 0$ *have? Answer this question not by algebra, but by graphing.*

**Brief Answer:** We see from the graph of $y = -x^2 + 4x - 5$ that there are no solutions to $0 = -x^2 + 4x - 5$.




**PRACTICE 3:**

a) *Find a value* $k$ *so that the graph of*

$$y = 5x^2 - 10x + k$$

*just touches the* $x$ *axis.*

*b) Find a value* $m$ *so that* $y = -2x^2 - 18x + m$ *gives the highest output value of* $100$.

*c) Find a value* $p$ *so that* $y = (x-p)(x-3p)$ *has smallest output value of* $-10$.

**Brief Answers:**

a) Writing $y = 5x(x-2) + k$ shows that $x = 1$ is the line of symmetry. We need the vertex of the graph of this equation to have height zero, so we need $0 = 5(1)(-1) + k$ meaning we need $k = 5$.

b) Write $y = -2x(x+9) + m$. We need $\left(-\frac{9}{2}, 100\right)$ to be the vertex, so we need $-2\left(-\frac{9}{2}\right)\left(\frac{9}{2}\right) + m = 100$ giving $m = \frac{119}{2}$.

c) The line of symmetry of the equation's graph is $x = 2p$. We need $(p)(-p) = -10$ so we need $p = \sqrt{10}$ or $-\sqrt{10}$.

**PRACTICE 4:** *Find, in terms of $r$, a value $k$ so that the graph of $y = 3x(2r - x) - k$ just touches the $x$ axis.*

**Brief Answer:** The line of symmetry is at $x = r$. We need $3(r)(r) - k = 0$ giving $k = 3r^2$.

**PRACTICE 5:** *Consider the equation*
$y = 2(x-1)(x+1) + (x-3)(x+4) + x(x-2)$

*a) Is this an equation in the form $y = ax^2 + bx + c$ in disguise for some numbers $a$, $b$, and $c$? (This is just a YES/NO question!)*

*b) Explain how one readily sees $a = 2+1+1 = 4$.*

*c) Explain why putting $x = 0$ into the right side of the equation allows us to conclude that $c = -14$.*

*d) What is the value of $b$?*

**Brief Answer:** a) YES. If we expand each product and collect like terms, we'd have an expression of the form given.
b) Expanding the products shows that

$$\begin{aligned} y &= 2x^2 + stuff + x^2 + stuff + x^2 + stuff \\ &= 4x^2 + stuff \end{aligned}$$

We see $a = 4$.

c) Putting $x = 0$ into $y = ax^2 + bx + c$ gives $y = c$. Putting $x = 0$ into the original expression gives
$y = 2(-1)(1) + (-3)(4) + 0 = -14$.
Thus $c = -14$.

d) We have so far

$$\begin{aligned} &2(x-1)(x+1) + (x-3)(x+4) + x(x-2) \\ &\quad = 4x^2 + bx - 14 \end{aligned}$$

Let's put in another value for $x$. And let's have the first product vanish by choosing $x = 1$.

$$0 + (-2)(5) + 1(-1) = 4 + b - 14$$

We see $b = -1$.

**PRACTICE 6:** *A rectangle has side lengths $7 - r$ and $3 + r$ for some value $r$. What value for $r$ gives a rectangle of maximal area?*

**Brief Answer:** The area is $A = (7-r)(3+r)$. This is a quadratic equation with $r = 7$ and $r = -3$ both interesting. The line of symmetry is at $r = 2$ and this is where the vertex of its graph lies. As this is a downward-facing quadratic, $r = 2$ gives the maximal value.

**PRACTICE 7:** *Here are three quadratic equations:*

(A) $y = 3(x-3)(x+5)$

(B) $y = 2x^2 + 6x + 8$

(C) $y = 2(x-4)^2 + 7$

*i) For which of these three expressions is it very easy to answer the question: "What is the smallest $y$-value the expression can produce?"*

*ii) For which of these three expressions is it very easy to answer the question: "Where does the graph of the equation cross the $x$-axis?"*

*iii) For which of these three expressions is it very easy to answer the question: "Where does the graph of the quadratic cross the $y$ axis?"*

*iv) For which of these three expressions is it very easy to answer the question: "What are the coordinates of the vertex in this equation's graph?"*

**Answers:** These answers are subjective.

i) (C) might be the easiest. We can see the smallest value of $7$ occurs for $x = 4$.

ii) (A) It crosses at $x = 3$ and $x = -5$.

iii) (B) Putting in $x = 0$ gives $y = 8$.

iv) (C) The vertex is $(4, 7)$.

**PRACTICE 8:** Here is a silly question.

*I make and sell BIPS. It costs me $80,000 + 200n$ rupees to make $n$ BIPS. If I sell $n$ BIPS I bring in $2n(600-n)$ rupees.*

*a) Ignoring costs, what number of BIPS sold brings in the largest number of rupees?*

*b) What number of BIPS should I sell to make the biggest profit?*

**Brief Answers:**

a) $2n(600-n)$ is a quadratic expression with $n = 0$ and $n = 600$ both giving a value of zero. The maximum value thus occurs for $n = 300$.

b) My profit is

$$\begin{aligned}&2n(600-n) - (80000 + 200n)\\&= -2n^2 + 1000n - 80000\\&= -2n(n-500) - 80000.\end{aligned}$$

We see $n = 0$ and $n = 500$ give symmetrical outputs and so the maximal profit occurs for $n = 250$.