## A 64 COIN MIND-READING TRICK 

Here's a surprising application of $1 \leftarrow 2$ machine codes (binary). Apparently, the flip of a single coin can impart enough information to specify any given cell of the 64 squares of a chessboard. This puzzle is sometimes called The Devil's Chessboard. It has been circulating through the mathematics community at least a decade.

The puzzle is challenging, and your students will no doubt feel daunted by it. Allow feeling overwhelmed as being part of the experience.

EXPLODING DOTS Topic:
Experiences 2: Having fun in base two.
Suggested Grade Level:
High School, and All.

## A 64 COIN MIND-READING TRICK

Here's a very tough puzzle. It's daunting and overwhelming and feels impossible to make headway on. First just read and try to make sense of what is being asked in the puzzle-and do feel overwhelmed. When ready, try thinking about the questions on the next page. These might help you lean into the challenge and start developing some thoughts on how to solve it. (But if frustrations feel too high at any point, feel free to go straight to the solution. Just working through the solution will be an enlightening experience.)

You and a colleague know you are soon to play the following "game." You each understand the rules of the game and you have some time to converse together about your strategy for possibly winning the game.

This evening, Mrs. X. will come fetch you, just you, and lead to a room. You will not see or be able to communicate with your colleague in any way from this point forward.

In the room is a single table on which sits an 8-by-8 chessboard and a pile of 64 identical coins.
Mrs. X. will place the coins on the board, one per cell, choosing at random as she goes along whether a particular coin sits heads up or tails up. You will be permitted to watch her do this.


When done, she will then point to a coin somewhere on the board and says: "This is the favored coin." Of course, there is nothing to distinguish one coin from another, but Mrs. X. will have you take note of one particular coin on the board.

Then Mrs. X. will have you flip one coin in the board: to change it from heads up to tails up, or vice versa. When done, you will then be taken away to wait alone in a separate room.

While you wait, your colleague will be brought into the room and simply told to pick up the favored coin. She may touch and pick up one coin only and it must be the correct favored coin.

What strategy can you both devise so that your colleague will know the correct coin to choose?
www.globalmathproject.org

## Some Possibly Helpful Questions:

1. Can you devise a strategy for a two-coin version of this puzzle?

Suppose Mrs. X. places two coins, either heads up or tails up, in a one-by-two board. She will point to either the left coin or the right coin as the favored coin.

There are only four possible layouts.


Could you and your colleague devise a strategy so that in each scenario, the flip a single coin could leave a picture that communicates either "left" or "right"?
2. Can you devise some strategies in certain cases for which Mrs. X.'s layout of heads and tails is not random?

For instance, if you and your colleague were told that Mrs. X. will set all 64 coins heads up, then it would be easy to communicate the favored coin: simply flip the favored coin and your colleague will see that one coin set to tails.

Suppose you and your colleague were told that Mrs. X. will layout a checkerboard pattern of heads and tails. Would flipping the favored coin work for your colleague in this case?

Or suppose you and your colleague were told that all but one 64 coins will be heads up, and that the favored coin will be among the heads. Can you devise a successful communication strategy in this scenario? (Would your strategy be ruined if Mrs. X. allowed to choose the single tails up coin as the favored coin?)

Can you devise other restrictions on how Mrs. X. might lay out the coins that readily lead to successful strategies?

## Some Overt Nudges and Hints:

1. It is suspicious that the number 64 is a power of two in a puzzle that might be making use of a $1 \leftarrow 2$ Exploding Dots machine.

Can we bring in binary codes of numbers?

It seems natural to number the cells of the chess board 1 through 64, say, starting at the top left cell of the board, reading left to right, top to bottom, so that that the bottom right cell is number 64.

Is it helpful to write the cell numbers in binary: 1, 10, 11, 100, 110, ...., 1000000?
2. The flip of a coin changes the coin from being among the set of "head coins" to the set of "tail coins", or vice versa. Is it helpful then to consider the set of all binary numbers that represent cells containing heads coins? We have the choice then to put in or take out a number from that set to communicate a specific cell number.

But how can a set of cell numbers communicate one cell number?

## THE PUZZLE EXPLAINED

It is actually better to number the cells 0 to 63 from the top left cell to the bottom right cell, and to represent these numbers as six-digit binary numbers. (Include leading zeros.)

000000, 000001, 000010, ...., 111110, 111111.
This feels "balanced" in that every six-digit sequence of 0 s and 1 s appears in this list. (Working with the numbers 1 through 64 is "unbalanced.")

Call a six-digit binary number a "heads number" if the coin in its cell position is heads. There could be anywhere from zero to sixty-four heads numbers.

From the set of heads numbers, create another six-digit binary number as follows:

1. Write out the heads numbers, one under the other.
2. Under each column, write a " 1 " if the count of 1 s in that column is odd and write a " 0 " if it is even. (If there are no heads numbers, that is, if Mrs. X. lays out nothing but tails, write 000000.)

For example, suppose Mrs. X. lays out heads on cells $2,11,17,22,46$, and 62 , and all other cells are tails. Then we create from this set the code 011110.


Now we have the opportunity to change this code by adding another heads coin to the set or to take one out. Can we change the code to match the cell number of the favored coin?

Yes!

Following the example, suppose the favored coin is in cell 53 , which has code 110101 . Then we need to change the parity (evenness/oddness) of four columns.


Consider the coin in the cell with the number that has a 1 in each of the columns we need to change and 0 in each of the columns we don't need to change. In our example, that's cell number 101011, which is 43.

If we add this coin to the list of heads coins (by turning coin 43 from tails to heads) or remove this coin from the list (by turning coin 43 from heads to tails), we change the count of 1 s in each column we need to change by one, and don't change the count of 1 s in the columns that we don't need to change. Either way, by flipping coin 43, we get a new set of heads coins that communicates the number of the coin in the favored cell!


In this way we can communicate to our colleague any desired cell number from any collection of heads coins Mrs. X lays out!

To summarize the procedure:

1. Look at the positions of the coins showing heads.
2. Write the cell number of each of these coins as a six-digit binary code from 000000 to 111111. Place these numbers in a vertical list.
3. For each column write 0 if the count 1 s in that column is even, write 1 otherwise. This gives an "auxiliary" six-digit code.
4. Compare the auxiliary code with the six-digit code of the cell number of the favored coin. For each position along that code, write 1 if there is a mismatch of digits, write 0 otherwise. This gives yet another six-digit code. This is the code of the cell number of the coin you flip. You've now arranged matters so that the heads on the board encode the cell number of the favored coin.
5. When your colleague enters the room, she conducts steps one, two, and three. She picks up the coin with cell number the auxiliary code she computes.

## Try it!

Try this game with the smaller example of a two-by-two board. Here the cell numbers are the two-digit binary codes $00,01,10$, and 11.


Does practicing with this small example help make sense of the procedure?

## EXTENSIONS

Every solved problem, of course, is an invitation to explore and play more. Might your students enjoy these explorations?

Wild Exploration: Is there a base six version of this puzzle? Suppose Mrs. X. laid a die on each cell of a board (what dimensions?), each showing one of the numbers $1,2,3,4,5$, and 6. (Perhaps regard " 6 " as the same as zero.) Could you change the showing face of one (or maybe five?) dice to communicate a particular cell of the board?

