CRYPTOGRAM 1 from BRILLIANT

Our partner BRILLIANT (www.brilliant.org) has a tremendous series of cryptogram puzzles—as part of an astonishing library of general puzzles and innovative mathematics content—that help students develop a profound and strong understanding of the standard arithmetic algorithms. Here are two puzzles from their *Practice Contest Math* series at https://brilliant.org/practice/whats-the-number/.

EXPLODING DOTS Topic:

Experience 3: Addition in a $1 \leftarrow 10$ machine.

Suggested Grade Level:
Upper ELEMENTRAY and MIDDLE SCHOOL

CRYPTOGRAM 1 from BRILLIANT

A cryptogram is a piece of arithmetic using standard algorithms—standard long addition or standard long multiplication, for instance—with some digits replaced by letters. If a letter is repeated, it means that the same digit appears in those places. Different letters represent different digits and no letter represents a digit you see unchanged.

When your students see this first example, they will likely immediately understand the challenge.

Puzzle 1 from https://brilliant.org/practice/whats-the-number/

$$\begin{array}{cccc} & & A & A \\ + & & 1 & A \\ \hline & & 8 & 2 \end{array}$$

What digit in place of A would make this sum true?

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Your students might simply start guess as to which digit A could be. It is clearly not 0, leaving it as either 3, 4, 5, 6, 7, or 9, the only digits not showing.

Of course, looking at the units column, it does first appear that A should be 1. But then one quickly realizes that the tens column is troubled. Allow your students to think through this, if they seem to be going that route. But do remind them afterwards that one of the features of the puzzle is that letters represent digits that aren't already showing.

Students will likely quickly realize that the ones column must add to 12 instead of 2, and this happens only for $A = 6$. All then does work out beautifully for the tens column too.

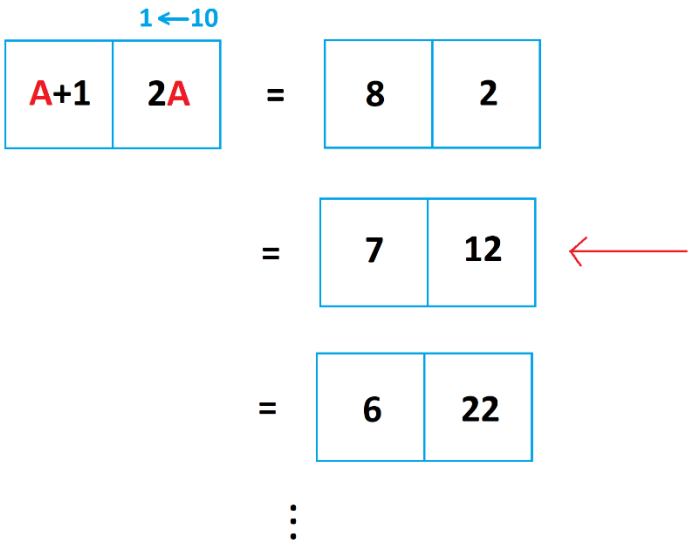
After solving this puzzle, suggest to students that they could alternatively think though this puzzle via the setup of a $1 \leftarrow 10$ machine.

Can they—and you—make sense of this rephrasing of the puzzle?

	1←10				
A+1	2A	=	8	2	

We can see right away that $A = 1$ does not work.

Let's play with explosions and unexplosions in the $1 \leftarrow 10$ machine and rewrite the right side.



In fact, after just one explosion, we see that the expression $7\,|\,12$ matches the left with $A = 6$.

Ask your students if they might like to try EXPLODING DOTS thinking on this next challenging question.

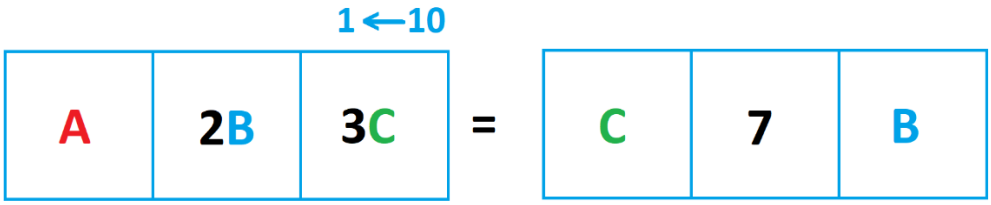
Puzzle 2 from https://brilliant.org/practice/whats-the-number/

$$\begin{array}{cccc} & A & B & C \\ & & B & C \\ + & & & C \\ \hline & C & 7 & B \end{array}$$

What is the value of the three-digit number $\overline{ABC}$?

Some Things Students Might Notice or Question

1. Your students will likely present the problem in a $1 \leftarrow 10$ machine as follows.

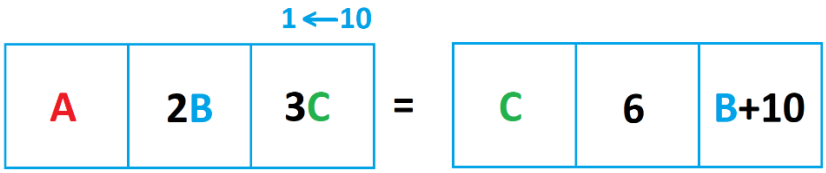


2. The entry $2B$ in the tens place on the left cannot directly match the entry 7 in the tens place on the right.

3. Comparing leftmost boxes, maybe $A = C$? But since they are meant to be distinct digits, this can't be the case. But how could it be otherwise?

4. Is $3C = B$?

5. Maybe we can unexplode one of the tens on the right to look at this picture instead?



   Could $B = 3$?

6. This is hard!

THE PUZZLE EXPLAINED

EXPERIENCE 3 of EXPLODING DOTS: Addition in a $1 \leftarrow 10$ machine.

Let's do look at the $1 \leftarrow 10$ machine presentation of the problem.

1←10

A	2B	3C	=	C	7	B

As A and C can't represent the same digit, there must at least one explosion in the tens place of the left picture that changes its leftmost box to a C. Let's do just one explosion for right now.

1←10

A+1	2B-10	3C	=	C	7	B

If we keep doing explosions from the tens place, the count of dots in that place will remain even ($2B$, $2B-10$, $2B-20$, …). So there must be some explosions from the units place as well. Let's do just one for now.

1←10

A+1	2B-9	3C-10	=	C	7	B

Actually, pause! If we now match the places, do the equations

$$A+1=C$$
$$2B-9=7$$
$$3C-10=B$$

yield an answer? Yes. We see $B=8$, then $C=6$, and then $A=5$, and this works!

$$\begin{array}{rrr} 5 & 8 & 6 \\ & 8 & 6 \\ + & & 6 \\ \hline = 6 & 7 & 8 \end{array}$$

If we have reason to believe there is only one solution to the puzzle, then we've found it! Woohoo! (Do we have reason to believe this?)

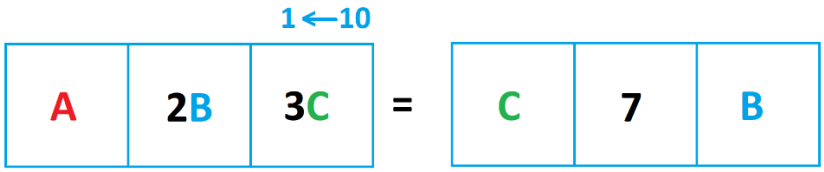
EXTENSIONS

Every solved problem, of course, is an invitation to explore and play more. Might your students enjoy these explorations?

Wild Exploration 1:

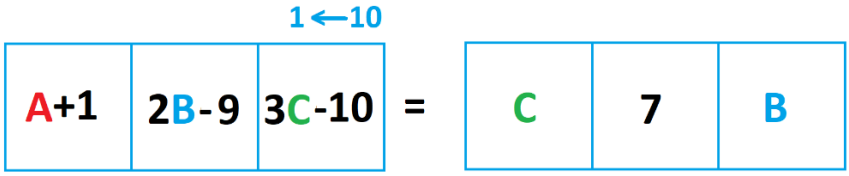
Develop some logic that explains why this is the only possible solution to the puzzle.

One approach might be to explain why there can at most one explosion from the tens place and at most one explosion from the units place in the left side of this picture.



Hint: *Each of B and C is a digit from 0 to 9. The largest possible value for $3C$ is 27, and so no more than two explosions can occur from the units place. What goes wrong with assuming it is two? The largest possible value of $2B$ is 18. (But some dots will also appear in that tens place via the explosions from $3C$. Does that produce worrisome effects?)*

Our work on the previous page shows that there needs to at least one explosion of each type, so this would establish that we must indeed be dealing with exactly one explosion in each box.



Wild Exploration 2:
There are other solutions to this puzzle if we don't require A, B, and C to be single digits. For example, $A = 7$, $B = 23$, and $C = 11$ works.

$$\begin{array}{rrrr} & 7 & 23 & 11 \\ & & 23 & 11 \\ + & & & 11 \\ \hline = & 7 & 46 & 33 \\ = & 7 & 47 & 23 \\ = & 11 & 7 & 23 \end{array}$$

Care to find other non-tradtional solutions?

Wild Exploration 3: Try the remaining puzzles from https://brilliant.org/practice/whats-the-number/.