DIVISIBILITY BY 37

We've explored the divisibility rules for **3** and **9**, and **7** in other essays, and in them we also paved the way for creating divisibility rules for **11**, **13**, **17**, **19**, and more.

In this essay we adopt a different approach for discovering more divisibility rules for awkward numbers. In particular, we present in detail a divisibility rule for the number **37**.

EXPLODING DOTS Topic:

Experience 6: Using dots and antidots for division in a $1 \leftarrow 10$ machine.

Suggested Grade Level:

Middle school. High school.

See Also:

Divisibility by 9
Divisibility by 9 again!
Divisibility by 7.

DIVISIBILITY BY 37

Do you and your students know this curious divisibility rule for the number 37?

> **To determine whether or not a number is divisible by 37, delete its first digit and add it to the digit three places further in. (Perform some carries, if necessary.) The original number is divisible by 37 only if this new number is.**
>
> **This procedure can be repeated until one has a number sufficiently small to recognize as a multiple of 37 or not.**

An example helps make sense of this. Let's check if 348022 is a multiple of 37.

$$\begin{array}{r} \cancel{3}48022 \\ +\quad 3 \\ \hline =\ 48322 \end{array} \qquad \begin{array}{r} \cancel{4}8322 \\ +\quad 4 \\ \hline =\ 8362 \end{array} \qquad \begin{array}{r} \cancel{8}362 \\ +\quad 8 \\ \hline =\ 370 \end{array}$$

As 370 is clearly a multiple of 37, the original number 348022 must have been too!

Check: $348022 = 37 \times 9406$.

As another example, this method shows that 3499 is not a multiple of 37: we recognize that $4\,|\,9\,|\,12 = 4\,|\,10\,|\,5 = 505$ is not a multiple of 37. (How?)

This algorithm is not very practical or helpful (are you able to readily tell if a three-digit number is a multiple of 37?). But the interesting part of this is not its practical use. We want to know:

Why does the algorithm work?

Some Things Students Might Notice or Ask

1. If you really want to know if a number is divisible by 37 or not, just get out a calculator! Clearly this is not a practical or efficient technique.

2. The algorithm is just weird.

3. It seems that that the algorithm doesn't actually give us the result of dividing a number by 37. It's just answering a YES/NO question.

SOLVING THE PUZZLE

See EXPERIENCE 6 of EXPLODING DOTS: Using Dots and AntiDots in Division

The algorithm relies on the fact that 999 is a multiple of 37. (It's 37×27.)
[This might be enough of a hint for your students to figure out what is going on on their own.]

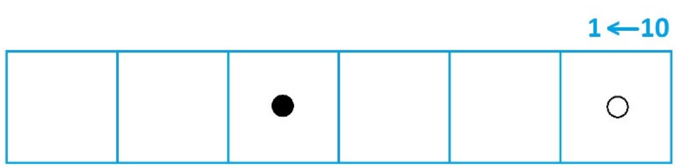
Let's look at numbers represented in $1 \leftarrow 10$ machines.

We want to assess whether or not, in identifying groups of 37, we'll see any remainders. No remainders yields a YES answer to our question (the number *is* divisibly by 37), some remainders yields a NO answer.

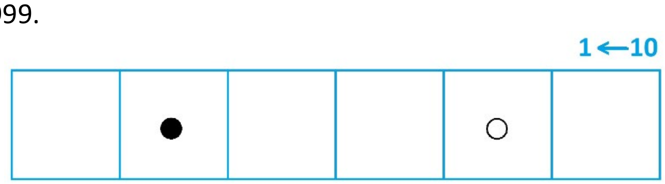
Since we don't care about how many groups of 37 we can find, we are free to add in extra groups of 37, or anti-groups of 37, in any way we like help answer our basic YES/NO question.

So, let's be clever! Let's work with groups of 999. (Each is a set of twenty-seven 37s.)

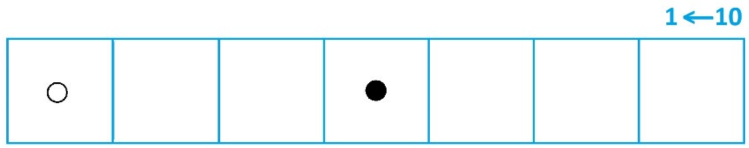
Think of 999 as $1000 - 1$. So here is one group of 999.



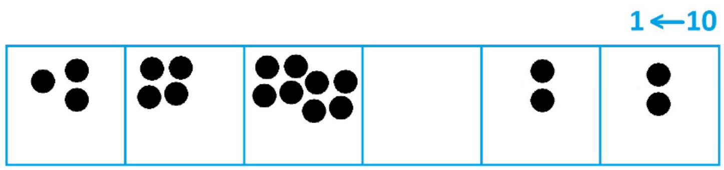
Here are ten groups of 999.



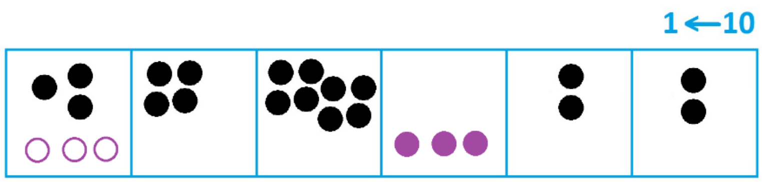
Here are one-thousand groups of “anti 999”.



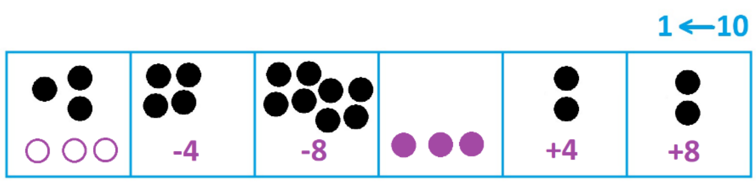
Given a number, such as 348022, in a $1 \leftarrow 10$ machine



let’s add some groups of anti 999. This won’t affect our analysis of whether or not finding groups of 37 leaves remainders. Let’s add them in such a way that deletes the first digit of the number!



Thus analyzing the divisibility of 348022 has been reduced to analyzing the divisibility of 48322. And in the same way this can reduced to analyzing 8362 and then to 370.



It is clear now how the divisibility rule works.

EXTENSIONS

Every solved problem, of course, is an invitation to explore and play more. Might your students enjoy these explorations?

Wild Exploration 1: The number 99999 is divisible by 271. What impractical divisibility rule can you construct from this?

Wild Exploration 2: The number 99 is divisible by 11. What divisibility rule can you create for the number 11 that first involves deleting the first digit of the given number. (How different is this rule from any rules for 11 you already know?)

Wild Exploration 3: You meet a Martian. It tells you, because they have three fingers on two hands, that they write all their numbers in base 6. But the Martian then claims that the *very same* divisibility rule for the number thirty-seven (which they write as "41") works in Martian: *Delete the first digit of the given number and add to the digit three places further in. Then the original number is divisible by "41" only if this new number is.* Is this true? Does the same divisibility rule for 37 work in Martian?

Wild Exploration 4: The number 1001 is divisible by 13. Use this to create a divisibility rule for the number 13 that starts by deleting the first digit of the given number.

Wild Exploration 5: Look at all the factors of 9, 99, 999, 9999, 99999, … and of 11, 101, 1001, 10001, 10001, … to create new divisibility rules that start by deleting the first digit of the given number.