THE FIVE-CARD MIND-READING TRICK

This is a classic number trick that serves as a perfect start to the entire EXPLODING DOTS story. Students just love this "mind-reading" trick and are fascinated by it. As soon as one's understanding of $1 \leftarrow 2$ machine codes come to light, the magic of this trick suddenly crystalizes. It's an AHA moment!

EXPLODING DOTS Topic:

Experiences 1 and 2: Understanding $1 \leftarrow 2$ machine (binary) codes.

Suggested Grade Level:

Upper elementary-, middle, and high-school

Our partner organization BRILLIANT also has plenty of materials that tie in beautifully with this topic. See the **Further Reading** at the end of the piece.

THE FIVE-CARD MIND-READING TRICK

Here's a classic trick. On a board write out the following five groups of numbers.

GROUP A	GROUP B	GROUP C	GROUP D	GROUP E
16 20 24 28	8 12 24 28	4 12 20 28	2 10 18 26	1 9 17 25
17 21 25 29	9 13 25 29	5 13 21 29	3 11 19 27	3 11 19 27
18 22 26 30	10 14 26 30	6 14 22 30	6 14 22 30	5 13 21 29
19 23 27 31	11 15 27 31	7 15 23 31	7 15 23 31	7 15 23 31

Now ask your students to each silently think of a number between 1 and 31. (They may choose the number 1 itself if they like or the number 31.)

Now perform the mindreading trick by having the following conversation with a student.

> "Suzzy. Is the number you are thinking of in group A?" "Yes."
> "Is the number you are thinking of in group B?"   "Yes."
> "Is it in group C?"   "No."
> "Is it in group D?"   "No."
> "Group E?"   "Yes!"
>
> "Ahh … your number is 25."   "Wow! Yes it is!"

Have the same conversation with a few more students, noting each time which groups elicit a "yes" answer from a given student. The secret number they have in mind is simply the sum of the top-left corner numbers in each group with a yes answer. For example, Suzzy answered YES YES NO NO YES. Groups A, B, and E have top left numbers 16, 8, and 1, respectively, and indeed $16 + 8 + 1 = 25$.

GROUP A	GROUP B	GROUP C	GROUP D	GROUP E
(16) 20 24 28	(8) 12 24 28	4 12 20 28	2 10 18 26	(1) 9 17 25
17 21 25 29	9 13 25 29	5 13 21 29	3 11 19 27	3 11 19 27
18 22 26 30	10 14 26 30	6 14 22 30	6 14 22 30	5 13 21 29
19 23 27 31	11 15 27 31	7 15 23 31	7 15 23 31	7 15 23 31
YES	**YES**	**NO**	**NO**	**YES**

Suzzy's responses for the number 25.

As practice, check that if Sameer is thinking the number 13, he will answer NO YES YES NO YES and indeed $8 + 4 + 1 = 13$.

Do this as many times as your students desire. Invite them to figure out what you are doing, and then, why what you are doing works! (Can you figure out why the top left numbers are key?) Perhaps facilitate thinking by writing a student's YES/NO answers under each group each time as done for Suzzy above.

Some Things Students Might Notice or Question

1. Students might notice that group E contains all the odd numbers.

2. Students might notice that group A contains all the numbers 16 and above.

3. Students might question the number 31. "Why must we choose between 1 and 31. What's special about the number 31?"

4. Students might notice the power of two in the top left corner of each group. (Though you might prefer to jumble the numbers in each group when you write them on the board to make this trick more mysterious!)

THE TRICK EXPLAINED

> See EXPERIENCES 1 and 2 of EXPLODING DOTS: Understanding $1 \leftarrow 2$ machine (binary) codes.

The key is to look at the $1 \leftarrow 2$ machine codes (that is, the binary codes) of the numbers from 1 up to 31.

$1 \leftarrow 2$

$$25 = $$

| 16 | 8 | 4 | 2 | 1 |
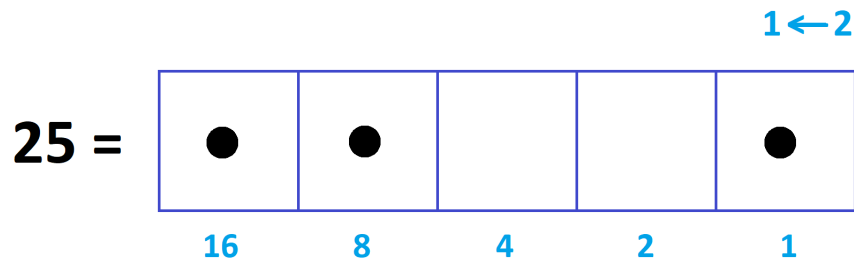
1: 1	9: 1001	17: 10001	25: 11001
2: 10	10: 1010	18: 10010	26: 11010
3: 11	11: 1011	19: 10011	27: 11011
4: 100	12: 1100	20: 10100	28: 11100
5: 101	13: 1101	21: 10101	29: 11101
6: 110	14: 1110	22: 10110	30: 11110
7: 111	15: 1111	23: 10111	31: 11111
8: 1000	16: 10000	24: 11000	

These are the codes with at most five digits. (The codes for the numbers 32 and above are six or more digits long.)

The numbers listed in group E are precisely the numbers with a 1 in the rightmost box for their $1 \leftarrow 2$ machine codes.

The numbers listed in group D are precisely the numbers with a 1 in the second-to-right box for their $1 \leftarrow 2$ machine codes.

The numbers listed in group C are precisely the numbers with a 1 in the third-to-right box for their $1 \leftarrow 2$ machine codes.

The numbers listed in group B are precisely the numbers with a 1 in the fourth-to-right box for their $1 \leftarrow 2$ machine codes.

The numbers listed in group A are precisely the numbers with a 1 in the fifth-to-right box for their $1 \leftarrow 2$ machine codes.

So, when a student answers "YES" to a particular group, she is informing you that there is a 1 in that matching box of her secret number's code. When she answers "NO," there is no 1.

For example, Suzzy informed us that the $1 \leftarrow 2$ code of her number is 11001. That the top left corner number in each group corresponds to the value of a dot in that group's matching box makes it quick to figure out which number has that code!

GROUP A	GROUP B	GROUP C	GROUP D	GROUP E
(16) 20 24 28	(8) 12 24 28	4 12 20 28	2 10 18 26	(1) 9 17 25
17 21 25 29	9 13 25 29	5 13 21 29	3 11 19 27	3 11 19 27
18 22 26 30	10 14 26 30	6 14 22 30	6 14 22 30	5 13 21 29
19 23 27 31	11 15 27 31	7 15 23 31	7 15 23 31	7 15 23 31
YES	YES	NO	NO	YES

$1 \leftarrow 2$

$$25 = \boxed{\bullet} \boxed{\bullet} \boxed{} \boxed{} \boxed{\bullet}$$

16   8   4   2   1
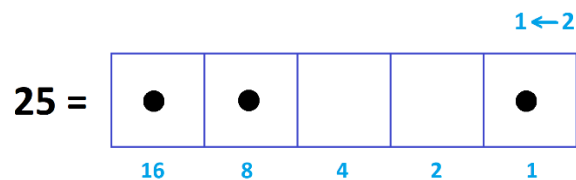
Question 1: Does it make sense that only the odd numbers have a 1 in the rightmost position for their $1 \leftarrow 2$ codes?

Question 2: Does it make sense that group E matches the numbers 16 through 31?

Question 3: Take the eight numbers in one group that are less than 16. Double them to obtain a new set of eight numbers. Now add one to each of those answers to obtain a second set of eight numbers. What do you notice about this collection of 16 numbers? Can you explain what you observe?

EXTENSIONS

Every solved problem, of course, is an invitation to explore and play more. Might your students enjoy these explorations?

Wild Exploration 1: What if we were willing to play with **six** groups of numbers? What would be a six-group version of this mind-reading trick? "Think of a number between 1 and ____"?

Wild Exploration 2: Here's a second mindreading trick. It works exactly the same way as the previous trick (all the mind-reader has to do is again look at the top left corner numbers to each group with a YES answer and sum those numbers to determine the secret number) but this time a participant is asked to think of a number between -10 and 21.

GROUP A	GROUP B	GROUP C	GROUP D	GROUP E
16 20 8 12	-8 -4 9 12	4 -4 20 12	-2 -10 14 6	1 5 9 15
17 21 9 13	-7 -3 8 13	5 -3 21 13	-1 -9 15 7	-1 -5 -9 17
14 18 6 10	-10 -6 6 10	2 -6 18 10	2 -6 18 10	3 7 11 19
15 19 7 11	-9 -5 7 11	3 -5 19 11	3 -5 19 11	-3 -7 13 21

Can you figure out why this trick works?

Further Reading:
Our partner BRILLIANT has a lovely series of puzzles based on the same ideas of this essay. Check out these four links.

BRILLIANT

https://brilliant.org/practice/how-binary-works/?chapter=introduction-40

https://brilliant.org/practice/exploding-dots/

https://brilliant.org/daily-problems/exploding-dots/

https://brilliant.org/practice/binary/?chapter=binary-and-other-bases-2