

MULTIPLICATION from BRILLIANT



Our partner BRILLIANT ([www.brilliant.org](https://brilliant.org)) has an astonishing library of general puzzles and innovative mathematics content. We start here with a warmup puzzle on long multiplication from Brilliant, and then push it further with the power of the *Exploding Dots* mindset! Our starting puzzle is item 2 from <https://brilliant.org/practice/calculation-tricks/?p=2>.



EXPLODING DOTS Topic:

Experience 3: Combining long multiplication with *Exploding Dots* thinking.

Suggested Grade Level:

Upper ELEMENTRAY and MIDDLE SCHOOL

MULTIPLICATION from BRILLIANT

Start by sharing this challenge from Brilliant.org with your students and asking if they can guess what the answer is going to be. Then have them confirm their guess by actually performing the long multiplication by hand.

Puzzle 2 from <https://brilliant.org/practice/calculation-tricks/?p=2>.

$$1 \times 1 = 1$$

$$11 \times 11 = 121$$

$$111 \times 111 = 12321$$

$$1111 \times 1111 = 1234321$$

Evaluate

$$11111 \times 11111.$$

We see that if the number that is a string of five 1s is multiplied by itself we get the answer 123454321.

$$\begin{array}{r}
 11111 \\
 \times 11111 \\
 \hline
 11111 \\
 111110 \\
 1111100 \\
 11111000 \\
 111110000 \\
 \hline
 = 123454321
 \end{array}$$

Ask your students:

What answer do you expect if the number that is a string of eight 1s is multiplied by itself?

$$11111111 \times 11111111 = ?$$

They will likely guess the answer 123456787654321 and can check this if they like by writing the long multiplication algorithm.



Now comes today's puzzle.

The number that is a string of ten 1s is multiplied by itself. What answer do we obtain?

$$1111111111 \times 1111111111 = ?$$

The number that is a sting of fourteen 1s is multiplied by itself. What answer do we obtain?

$$11111111111111 \times 11111111111111 = ?$$

Some Things Students Might Notice or Question

1. Some students might wish to write out the long multiplications. That's okay.
2. Students will likely comment that for the first product the middle digit needs to be 10, and 14 for the second product.
3. Students who have followed *Exploding Dots* might suggest we can perform explosions on these digits.

THE PUZZLE EXPLAINED

EXPERIENCE 3 of EXPLODING DOTS: Combining long multiplication with Exploding Dots thinking.

Multiplying a ten-digit string of 1s with itself does gives the answer

$$1|2|3|4|5|6|7|8|9|10|9|8|7|6|5|4|3|2|1$$

if we follow the *Exploding Dots* mindset and don't worry about carrying. (Having performed smaller examples, this answer will likely be clear to many students. But performing the long multiplication to be certain, of course, does not hurt.)

With explosions, this transforms to the answer 1234567900987654321.

Multiplying a fourteen-digit string of 1s with itself gives the answer

$$1|2|3|4|5|6|7|8|9|10|11|12|13|14|13|12|11|10|9|8|7|6|5|4|3|2|1.$$

With explosions this transforms to the answer 123456790123454320987654321.



EXTENSIONS

Every solved problem, of course, is an invitation to explore and play more. Might your students enjoy these explorations?

Wild Exploration 1:

The computation 1×1 gives an answer that is 1 digit long.

The computation 11×11 gives an answer that is 3 digits long.

The computation 111×111 gives an answer that is 5 digits long.

The computation $111111111 \times 111111111$ (with each number composed of ten 1s) gives an answer 19 digits long.

In computing $111111111111111111 \times 111111111111111111$ (each number has twenty 1s) how many digits do you expect in the answer?

[Warning: Might explosions work their way all the way to the left and create an extra digit you don't first expect?]

Wild Exploration 2:

Do any these sets of products (or other that you might choose to play with) have interesting features?

$$2 \times 2$$

$$2 \times 1$$

$$9 \times 9$$

$$22 \times 22$$

$$22 \times 11$$

$$99 \times 99$$

$$222 \times 222$$

$$222 \times 111$$

$$999 \times 999$$

$$2222 \times 2222$$

$$2222 \times 1111$$

$$9999 \times 9999$$

$$\vdots$$
$$\vdots$$
$$\vdots$$

Wild Exploration 3:

Explore these sets of products. What do you notice?

$$11$$

$$101$$

$$1001$$

$$11 \times 11$$

$$101 \times 101$$

$$1001 \times 1001$$

$$11 \times 11 \times 11$$

$$101 \times 101 \times 101$$

$$1001 \times 1001 \times 1001$$

$$11 \times 11 \times 11 \times 11$$

$$101 \times 101 \times 101 \times 101$$

$$1001 \times 1001 \times 1001 \times 1001$$

$$\vdots$$
$$\vdots$$
$$\vdots$$

Wild Exploration 4: Try the remaining puzzles from <https://brilliant.org/practice/calculation-tricks/?p=2..>