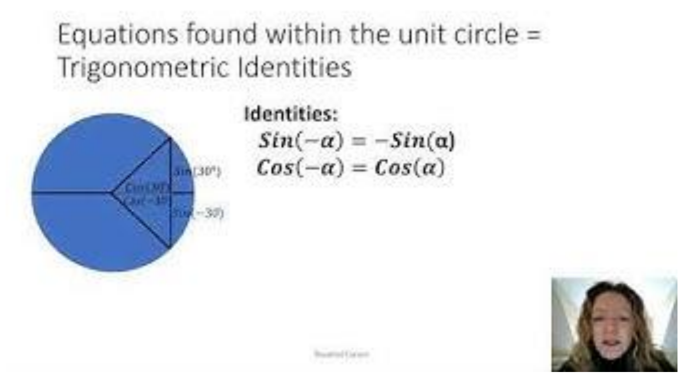
The New Perspective for Teaching/Learning Trigonometry!
(Well, it's a return to the old!)

Lesson 2: Exploring Relationships Within Sine and Cosine Values

TEACHER NOTES

The Student handout that follows is with regard to my video showing the nice relationships between sine and cosine values.



In this lesson you and your students will:

- Visually discover the cosine value (distance from centre) of a positive angle movement is equal to the cosine value of the same angle rotated in a negative direction.
- Visually discover the sine value of a negative angle movement is equal to the negative value of sine for the angle in a positive rotation.
- Geometrically see the equivalence relationships between sine and cosine values using a rectangle!
- The relations between sine and cosine values can also be observed from the table of values and graphs explored in video Tig Lesson 1.1.

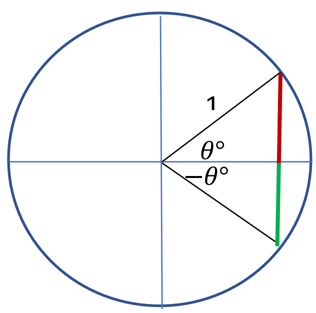
This lesson will require:

- Students to draw their own circles and angle measurements to OBSERVE the SINE and COSINE relationships hold true for their OWN constructions.
- Students can access their tables of values and graphs previously constructed from Trig Lesson 1.1. Their work can be referred to when the teacher facilitates a class discussion highlighting the relationships of sine and cosine from this lesson: Trig Lesson 1.2.

Lesson 2: Sine and Cosine Relations

Equipment: Compass, Ruler, Pencil, Printed Handout, and Math Curiosity

1. Watch Trig Lesson 1.2 and stop video at 6min 10 sec mark.
2. Construct a circle, use a ruler to draw in *x* and *y* axis (point (0,0) at centre of circle). Rotate the radius a specific amount of degrees counter-clockwise from the horizon (x-axis).
3. Draw in the radius location after the negative rotation of the same degrees (clockwise rotation).
4. Draw in the chord length connecting the endpoints of the 2 radii drawn in step 2 and 3.
5. Label the half-chord length with respect to the positive $\theta°$.
6. Label the half-chord length with respect to the negative rotation of that same degree movement ($-\theta°$).



7. After completing the above constructions, try to fill in the following values:

$\theta°$	Sin($\theta°$)	Cos($\theta°$)	A place to sketch:
30°			
-30°			
330°			
-330°			
45°			
-45°			
315°			
-315°			
150°			
-150°			

- What is the same about some of the values in the chart?

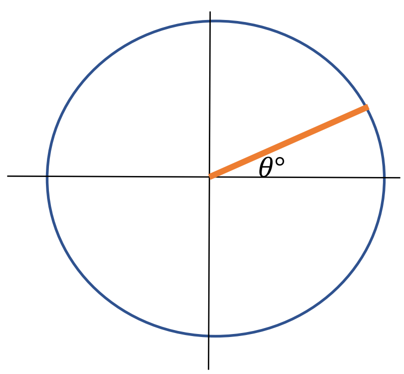
- What is different about some of the values in the chart?

- Can you describe any patterns in terms of reflections?

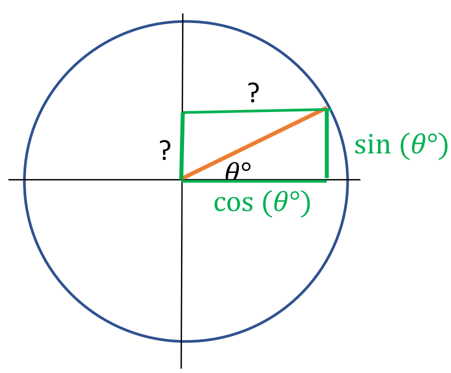
- What are the trigonometric patterns that emerge from the chart? Can you write an equation(s) that describe those patterns?

Equations involving trigonometric functions that describe patterns and hold true for any angle movement are called *Trigonometric Identities*.

8. Watch the video from 6min 45sec mark.
9. Construct a circle with a radius located in Quadrant I. See the following diagram:



10. Label in the half-chord length and the distance from centre for $\theta°$.
11. Draw in the rectangle with the side lengths $\sin(\theta°)$ and $\cos(\theta°)$.



12. Find the remaining two side lengths of the rectangle. Try to use a different angle other than $\theta°$ to label the remaining sides of the rectangle using $opp = hyp \cdot sin$ and $adj = hyp \cdot cos$.
13. Opposite sides in a rectangle are of equal length, producing an interesting relationship between sine and cosine values that are a $90°$ apart. This phenomenon can also be observed in the graphs of sine and cosine. It is also possible to have discovered this relationship between sine and cosine from the table of values generated in Trig Lesson 1.1 (exact values of Sine and Cosine in increments of $30°$ *and* $45°$).

Play around with some angles and see if the equations for sine and cosine relations will hold true! These are two more trigonometric identities to add to the formula sheet.

$\theta°$	$\sin(\theta°)$	$\cos(90 - \theta°)$	$\cos(\theta°)$	$\sin(90° - \theta°)$
$0°$				
$30°$				
$45°$				
$60°$				
$90°$				
$120°$				
$-30°$				
$-45°$				

Trigonometric Identities from Lesson 1.1 and 1.2:

- $\boldsymbol{Sin(90 - \theta) = Cos(\theta)}$
- $\boldsymbol{Cos(90 - \theta) = Sin(\theta)}$
- $\boldsymbol{Sin(-\theta) = -Sin(\theta)}$
- $\boldsymbol{Cos(-\theta) = Cos(\theta)}$
- $\boldsymbol{Sin^2(\theta) + Cos^2(\theta) = 1}$
- $\boldsymbol{Tan(\theta) = \frac{Sin(\theta)}{Cos(\theta)}}$