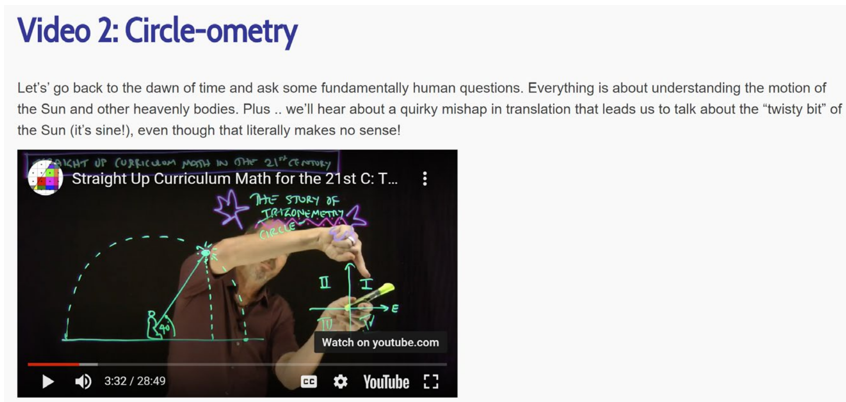
## The New Perspective for Teaching/Learning Trigonometry
(Well, it's a return to the old!)

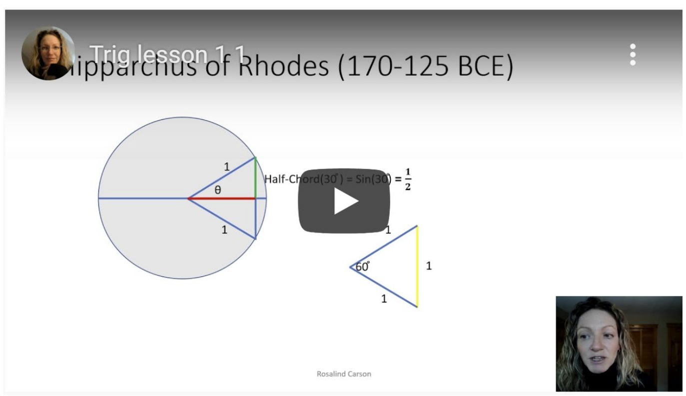
### Lesson 1: CIRCLE-OMETRY

# TEACHER NOTES

The student handout that follows is with regard to James Tanton's video on circle-ometry video, the first experience for students on the topic of trigonometry. Watch James' video right now if you have not yet seen it.




Also, review my video as I give some additional concrete advice on implementing this content.




### In this lesson you and your students will:

- Discuss the historical roots of trigonometry
- Compute the sine and cosine of angles of elevation related to a 90-degree angle.
- Compute the sine and cosine of angles of elevation related to a 45-degree angle.
- Compute the sine and cosine of angles of elevation related to 30- and 60-degree angles.
- Review of properties of special triangles and the Pythagorean Theorem will naturally arise.

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### The lesson will require:

- Having a relaxed and playful discussion with your students.
- A possible need to discuss characteristics of different types of triangles: right isosceles triangles and equilateral triangles will emerge from the discussion. Discuss properties of specific triangles as the triangles arise. (Don't fall into the pedagogy trap of giving students content before it is needed simply because "they will need to know it later.")
- Need of the fact that the sum of the three interior angles of any triangle equals 180°.
- Need of the Pythagorean Theorem for right triangles.

### Conducting the lesson:

1. Watch with your students or have your student watch James' video.
2. Discuss and emphasize the key points of the story.

   *Why are we working with a circle of radius 1?*

   *Why are angles in mathematics always measured from the right side of a horizontal axis?*

   *Why do you think clocks go clockwise but math prefers the counter-clockwise direction?*

   *What are the names of the height and overness of the Sun? Why those funny names?*
3. Ask your students to complete the questions and the two tables in the handout—even though James did some of the computations in the video. Ask your students to see if they can reconstruct the reasoning for themselves. (After all, they are in ones in charge of their own learning and understanding.)
4. While some students are filling in the chart and looking for patterns, other students can be graphing the half-chord lengths as a function of degree movement, $y = \sin(x)$, and also plotting the data to graph the distance of the chord from the center of the circle, $y = \cos(x)$. All students can be looking for patterns in the data and in the graphs and can share in a class discussion.

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# STUDENT HANDOUT

1. Watch James' video here. You, your classmates, and your teacher will discuss the key points of the story.

   Here are some questions that attend to key points in the story. Are there other questions you could ask to help make sure you and your colleagues really "get it"?

   *Why are we working with a circle of radius 1?*

   *Why are angles in mathematics always measured from the right side of a horizontal axis?*

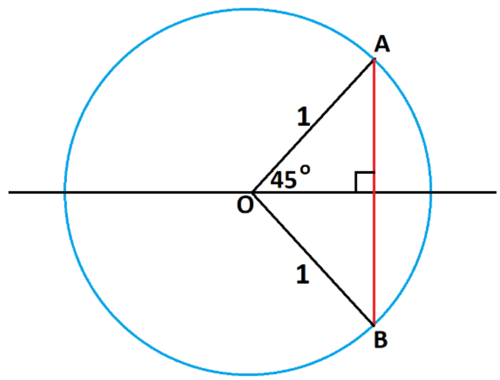
   *Why do you think clocks go clockwise but math prefers the counter-clockwise direction?*

   *What are the names of the height and overness of the Sun? Why those funny names?*

2. Complete the following table.
   (James did some of these in the video, but can you reconstruct the value for yourself?)

| Degrees ( $\theta°$) | $sin(\theta°)$ | $cos\ (\theta°)$ |
|---|---|---|
| $0°$ | | |
| $45°$ | | |
| $90°$ | | |
| $135°$ | | |
| $180°$ | | |
| $225°$ | | |
| $270°$ | | |
| $315°$ | | |
| $360°$ | | |
| $-45°$ | | |

3. The following picture shows that we can literally think of $sin(45°)$ as half the length of the red chord.




a) What type of triangle is $\Delta OAB$ ?

b) What is the length $AB$ ?

c) What, again, is the value of $\sin\left(45^{\circ}\right)$ ?

d) In his video James obtained $\sin\left(45^{\circ}\right)=\frac{1}{\sqrt{2}}$. In part c) you likely obtained $\sin\left(45^{\circ}\right)=\frac{\sqrt{2}}{2}$. Are these indeed equivalent values?

4. Fill in the chart of lengths as you travel around the circle. (Just copy in your answers from question 2 in the appropriate rows.) If you are feeling game, you could use this data to start plotting a graph of $y = \sin(x)$ and a graph of $y = \cos(x)$.

| Degrees ( $\theta°$) | $sin(\theta°)$ | $cos\ (\theta°)$ |
|---|---|---|
| $0°$ | | |
| $30°$ | | |
| $45°$ | | |
| $60°$ | | |
| $90°$ | | |
| $120°$ | | |
| $135°$ | | |
| $150°$ | | |
| $180°$ | | |
| $210°$ | | |
| $225°$ | | |
| $240°$ | | |
| $270°$ | | |
| $300°$ | | |
| $315°$ | | |
| $330°$ | | |
| $360°$ | | |
| $390°$ | | |
| $-45°$ | | |
| $-30°$ | | |