TRADITIONAL CURRICULUM TRIGONOMETRY

Here's an approach to traditional SOHCAHTOA that perhaps aligns with the "circle-ometry" (aka, the unit circle) students will later explore.

Scaling in Geometry

Here’s an idea of an activity for students relevant to the mathematics needed.

ACTIVITY: Open a street map on a smart phone.

1. Take note of some right angles you see between streets and estimate the angles at which some other streets intersect.

   Now use your fingers to enlarge the map.

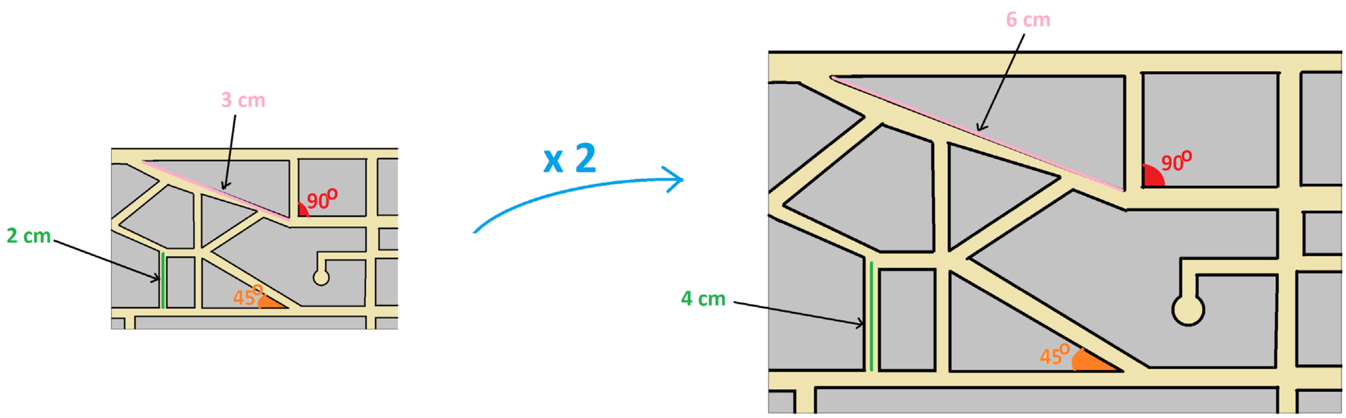
   How do the angles between streets change?

2. Scale the map down to roughly the scale it was before. With a ruler measure three or four block lengths or street lengths.

   Now finger-swipe again to enlarge the map. Remeasure the same block and street lengths. What do you notice?

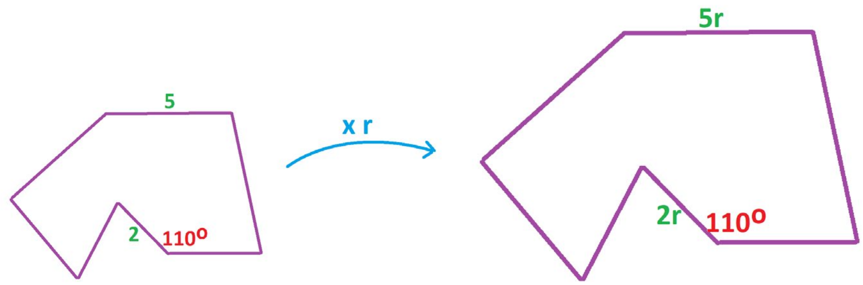
The goal is to help students see that if we reduce or enlarge geometric figures or diagrams

- All angles in the diagram remain unchanged in measure
- All lengths change by a consistent factor (the **scale factor** of the enlargement).



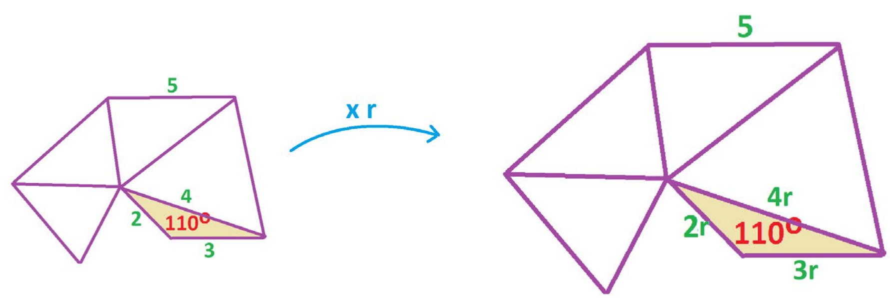
The simplest figures to study in geometry are straight lines and the shapes formed by straight line segments: polygons.

Two polygons are said to be **similar** if one is a scaled copy of the other.



This means, that corresponding angles in the polygons match in measure exactly, and corresponding sides are scaled by a fixed factor.

Actually, since any polygon can be divided into triangles, we can focus just on how triangles scale and ask what makes two triangles similar. (Studying just triangles is FAR LESS work than studying all polygons. I am up for that!)



ACTIVITY: Have each student in your class to draw a triangle possessing one angle of measure of 15° and one angle of measure 90°. (What must the measure of the third angle be?)

How different are everyone's pictures? What do notice about their side lengths? (Use rulers to measure side lengths.)

Lead students to notice that, for any two chosen drawings, the triangles have side lengths in matching scale. That is, all triangles drawn are similar.

SOH, CAH, and then TOA

It is a fundamental belief in geometry that **if two triangles have two angles that match in measure, then those triangles are scaled copies of each other.** (This is usually called the *AA principle.)*

Presumably this axiom of geometry has already been discussed and explored as an earlier curriculum unit. Review and explore as needed, perhaps expanding on the activities here.

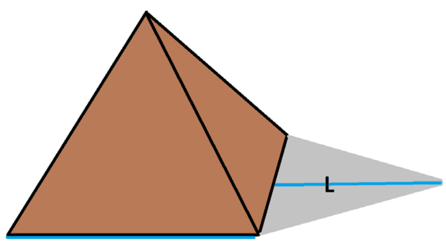
A Historic Feat

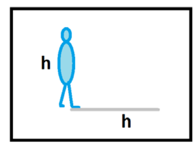
Here's a discussion item for your class. Draw diagrams on your board or a projector or share the first handout with your students.

Thales of Ionia (640-546 BCE) garnered tremendous fame as a geometer by deducing the height of the Great Pyramid of Egypt simply by measuring shadow lengths.

He waited for the day of the year that the Sun set directly behind the pyramid and also waited for the exact time of day that his body cast a shadow the same length as his height. Then, at that very moment, he measured the length L of the shadow shown. As it is easy to measure the base width B of the Great Pyramid, Thales had all the information he needed to deduce its height.

Can you see how? Can you explain the mathematics behind his method?

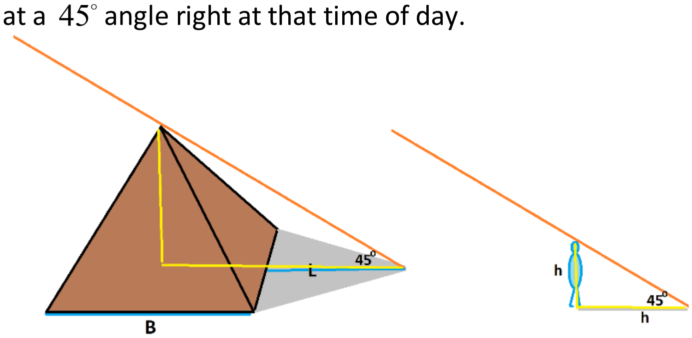




The base length of the pyramid is $B = 230$ m. Suppose the length of the shadow at his special time of the special day is $L = 32$ m. What is the height of the pyramid?

Help your students realise and articulate all the pieces here.

- Given the vast distance of the Sun and the large curvature of the Earth, all the rays of light from the Sun intercept the ground at essentially the same angle when standing in the vicinity of the pyramid.
- Thales' and his shadow form a right isosceles triangle. The rays from the Sun must be coming in at a 45° angle right at that time of day.



- Thus, the height of the Great Pyramid and the length $\frac{B}{2} + L$ form a right isosceles triangle too. These two side lengths must be of the same measure.

- The height of the Great Pyramid is $\frac{B}{2}+L$. Since he can measure B and measure L he can determine this quantity.

Question: Are we invoking the AA similarity principle here?

(Answer: It doesn't seem so. We're just invoking the fact that the large triangle has a right angle and a two 45° angles, making it isosceles, thus the height matches the length along the ground, which we can measure.

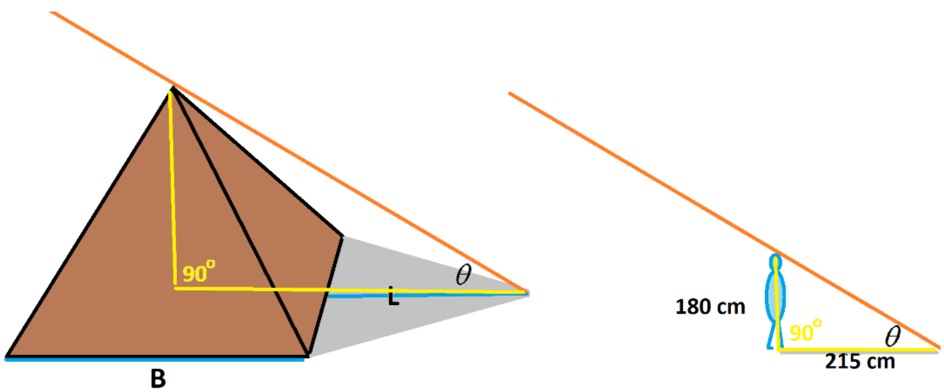
But proving the claim, "If a triangle has two angles equal in measure, it must be isosceles," does require use of the AA principle!)

Pushing this Idea Further

Ask:

Did Thales really need to wait for the exact time of day when his shadow length matched his height?

Suppose Thales was 180 cm tall and he conducted this exercise at a time of day when his shadow was 215 cm long. Could he still have determined the height of the Great Pyramid?



Conduct a discussion (whole class, small groups, pairs – as you see fit) and help students realise

- Because the Sun's rays intercept the ground at (essentially) the same angle, we have two triangles with two matching angles. We have similar triangles.
- Consequently, matching sides are scaled by the same factor.
- Because Thales' can measure B and measure L he knows what the scale factor is by comparing $\frac{B}{2}+L$ with the number 215.
- The height of the pyramid is the number 180 cm adjusted by this scale factor.

Now add

The base length of the pyramid is $B = 230$ m.

Given that you do already know the height of the pyramid from the previous exercise (147 m), what is the length L of the shadow in this scenario?

Help students come to the answer $L \approx 32$ m.

We have $\frac{B}{2} + L = 115 + L$ meters, and so $115 + L \; = \; 2.15r$ and $147 = 1.80r$ for some scale factor r. (We've written all measurements in terms of meters.) Thus $r = \frac{147}{1.80}$ and

$L = 2.15 \times \frac{147}{1.80} - 115 \approx 60.6$ meters.

How tall is a flagpole?

ACTIVITY

On a sunny day head out to a plaza or a field with flagpole equipped with a tape measure and a friend. Have your friend stand tall. Measure the length of her or his shadow. Measure the length of the shadow of the pole.

Do you have all the information you need to find the height of the pole? (If not, measure something in addition!)

EXTRA:

Students are unlikely to obtain the same answer. Why? The effect of measurement error!

Perhaps conduct some discussion here about this.

How exact were you in measuring Suzzy's height? To the nearest centimeter? Half a centimeter? The length of her shadow? The length of the pole's shadow?

Give some error ranges for your measurements.

Example:

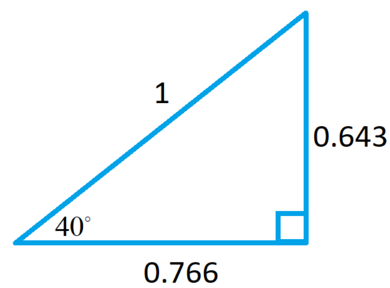
Suzzy's height: 148 ± 1 cm
Suzzy's shadow: 64 ± 1 cm
Flagpole shadow: 255 ± 5 cm

Given your ranges of values, what's the shortest height the flagpole could be? The highest it could be?

Getting More Abstract

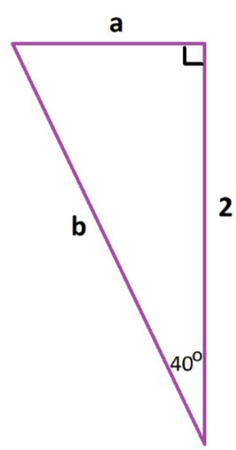
A question to guide class discussion.

Here are the side lengths of a right triangle possessing an angle of 40°.



(These numbers were found by drawing such a right triangle with a hypotenuse 1 and measuring the two side-lengths by hand. Measurements were conducted to three decimal places.)

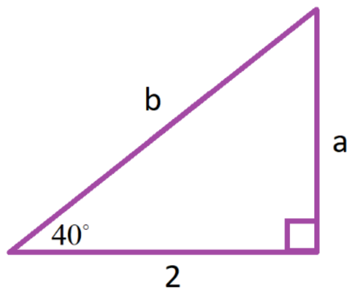
Can you now determine the lengths a and b in this figure?



Help your students come to realise that there is enough information here to deduce what the scale factor between the two figures must be.

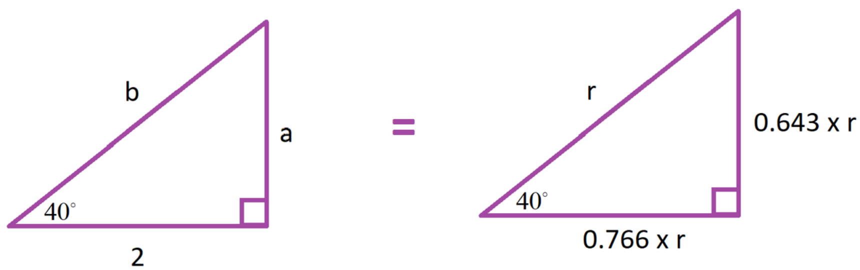
Go through the following specific pointers:

1. It could help to redraw the purple triangle so that its orientation is less befuddling.



2. This is a scaled copy of the given basic right triangle. It is helpful to write in the scaled values of the basic right triangle.

   If we scale the basic triangle with side lengths 1, 0.643 and 0.766 by a factor r, we obtain a triangle with side length r, $0.643r$, and $0.766r$.



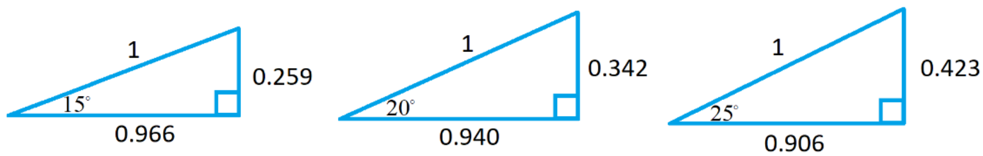
3. We can now see what the value of r must be (it's $\frac{2}{0.766}$) and hence deduce that

   $a = 0.643 \times \frac{2}{0.766} = 1.679$ and $b = 1 \times \frac{2}{0.766} = 2.611$.

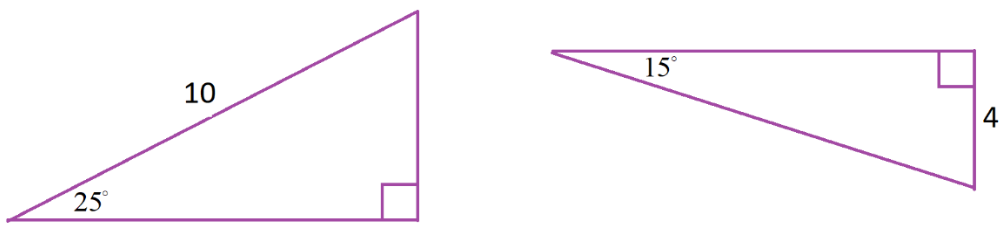
Offer your students the following practice problems.

Some Practice

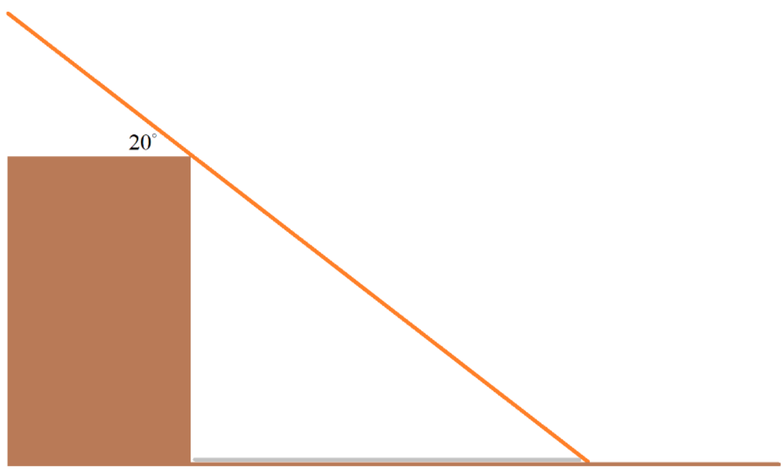
Here the side lengths of three right triangles each with hypotenuse 1. Use them for the following two questions.



Question 1: Find the two missing side lengths in each of these right triangles.

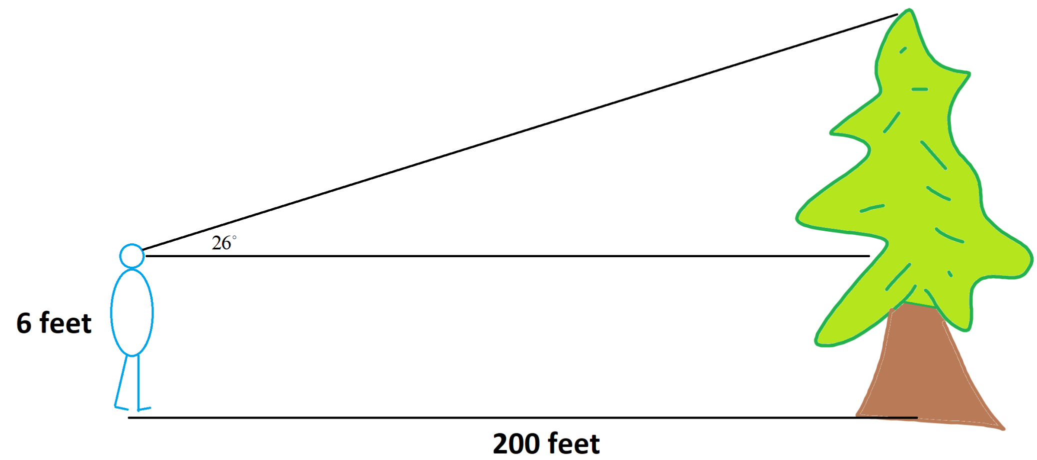


Question 2: A cliff casts a shadow 200 meters long when the rays of Sun hit the surface of the Earth at an angle of 20°. How high is the cliff?



(Scientists use essentially this method to determine the depth of craters on the Moon by looking at the lengths of shadows on surface photographs and knowing the angle of the Sun at the time the photographs were taken.)

Question 3: Estimate the height of the tree.



SOH, CAH, and then TOA

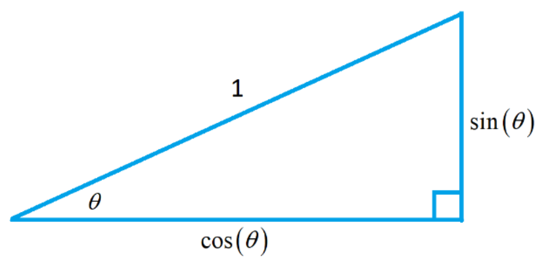
Continuing the Discussion

Share the following with your students.

Humankind since the time of Thales has realised that knowing the side lengths of basic right triangles can be of immense practical use in matters of architecture, engineering, geography, navigation, and such. Just by measuring angles and knowing only one length it is possible to deduce additional lengths- be they heights of pyramids or trees, or distances to objects, or depths of craters.

So, it became a study of great interest to know the side-lengths of basic right triangles. For historical reasons, folk worked with right triangles of hypotenuse 1.

If a right triangle contains an angle θ, then the side lengths of the triangle are called *sine* and *cosine* and are denoted $\sin(\theta)$ and $\cos(\theta)$. They are the sides shown in this diagram.

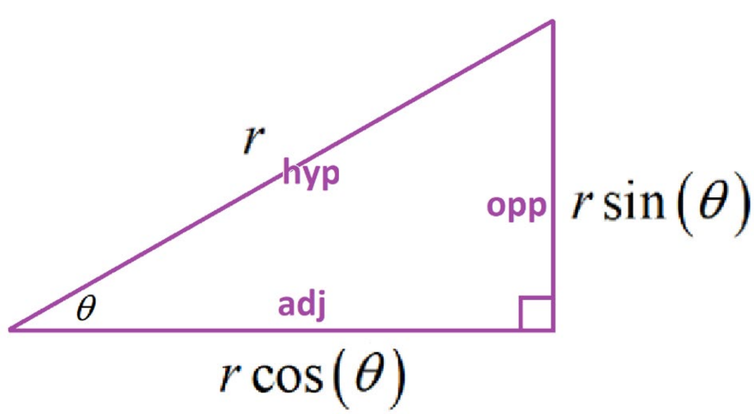


These are strange names and the story that led us to them is fascinating. Watch this video up to the 13:43 mark to see this history (and afterwards you will have no trouble remembering the diagram above: "sine" is height and cosine is "overness.")

Your calculator has been programmed to give you these sine and cosine values for any angle θ. (Oh, how scholars of ancient times yearned for the means to know these values so readily!)

The key thing to remember is that

Any right triangle with the same angle θ is a scaled copy of this basic right triangle and so must be of the following form for some value r.



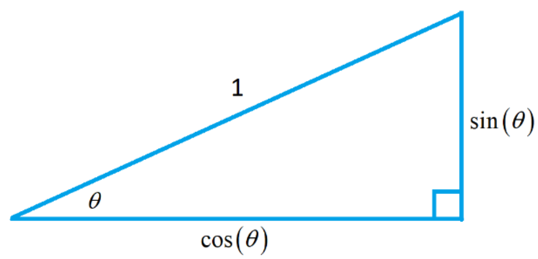
To be explicit:

The hypotenuse (hyp) will have a value r.

The side opposite the given angle θ (opp) will be $r\sin(\theta)$.

The side adjacent to the given angle θ different from the hypotenuse (adj) will be $r\cos(\theta)$.

Better yet, just keep the image of a scaled copy of the basic triangle in one's mind.



For example, a calculator gives

$$\sin(23^\circ) = 0.391$$
$$\cos(23^\circ) = 0.921$$

and so a right triangle with possessing an angle of 23° and a hypotenuse of length $r = 100$ has sides of lengths scaled up by 100.

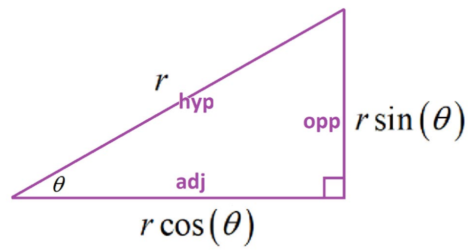
$$opp = r\sin(23^\circ) = 39.1$$
$$adj = r\cos(23^\circ) = 92.1.$$

Some more practice problems.

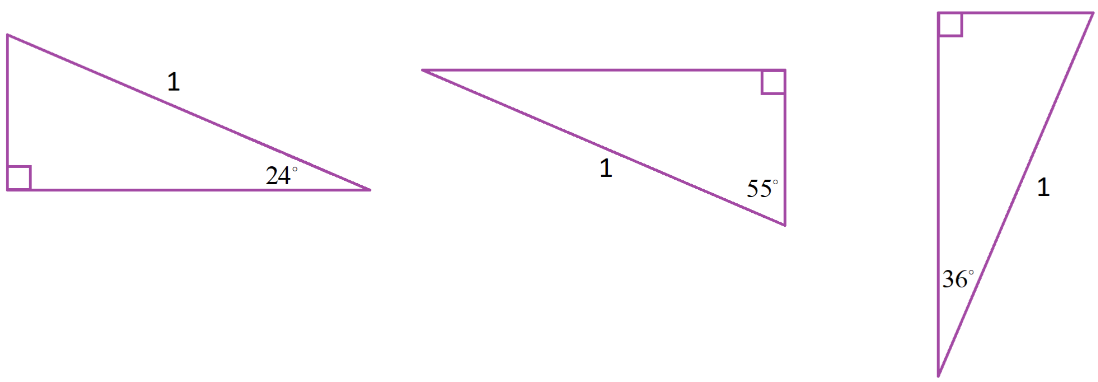
More Practice

For a right triangle possessing an angle θ we have

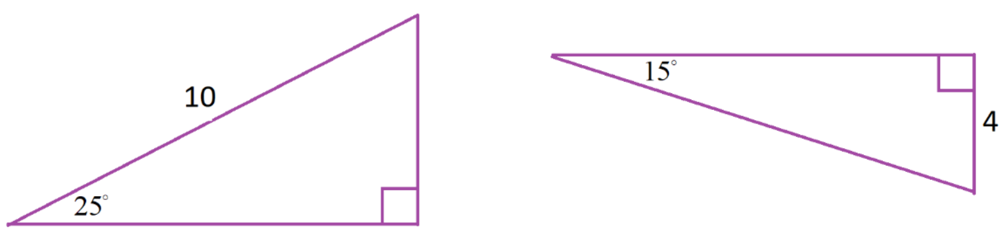
$$hyp = r$$
$$opp = r\sin(\theta)$$
$$adj = r\cos(\theta)$$



Question 1: According to your calculator, to three decimal places, what are the two missing side lengths in each of these triangles?



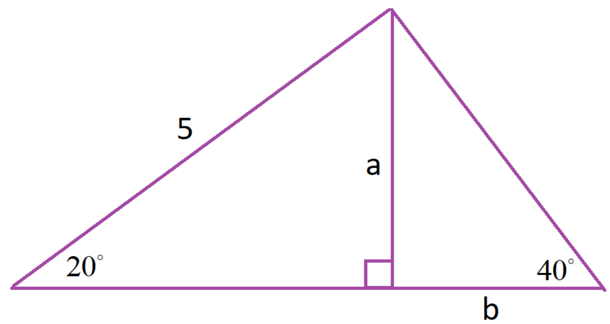
Question 2: Find the two missing side lengths in each of these right triangles.



SOH, CAH, and then TOA

Next are some more challenging practice problems. You might like to work with one of the problems in the set with the class as a whole first. This one:

Find the values of lengths a and b in this figure. Round your answers to three decimal places.



Any first thoughts on how to get started?

Help your students reason as follows.

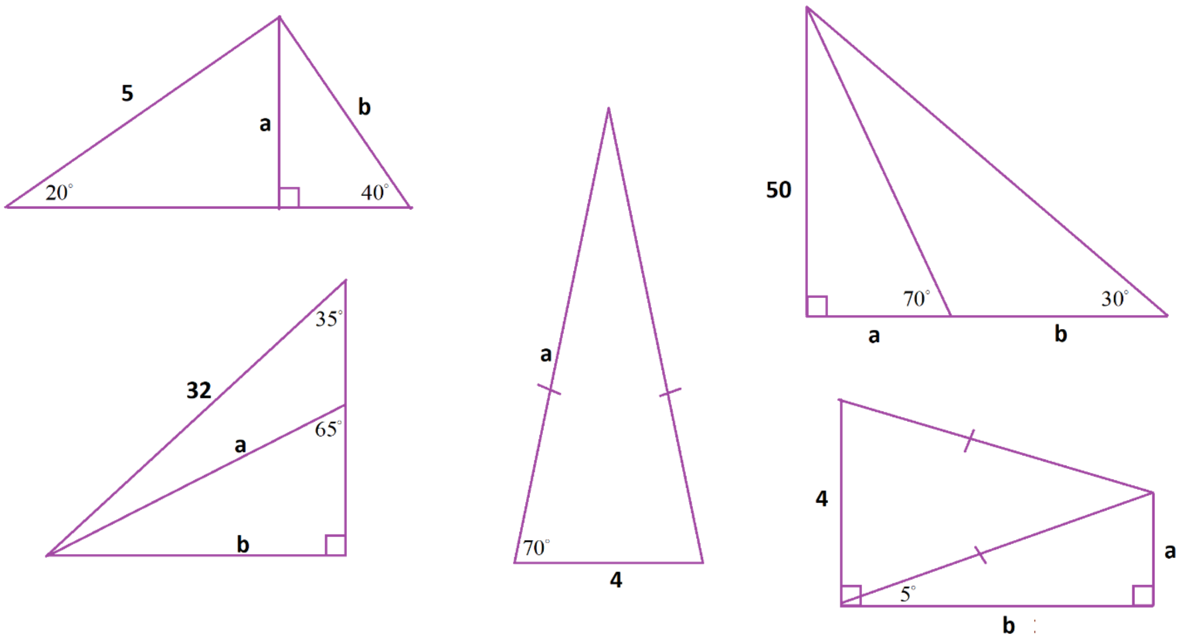
- In the right triangle with angle 20°, we have hypotenuse 5 and a is the side opposite angle 20°. Thus $a = 5\sin\left(20^\circ\right) = 1.710$, to three decimal places.
- In the right triangle with angle 40°, we don't know the value of the hypotenuse r, but we do know that $a = r\sin\left(40^\circ\right)$.

  So $1.710 = r\sin\left(40^\circ\right)$ and this gives $r = 2.660$.
- Then it follows that $b = r\cos\left(40^\circ\right) = 2.660 \times 0.766 = 2.038$

Answers may vary in the latter decimal place(s) depending on whether rounding is conducted with intermediate values along the way or left as a final step.

Yet More Practice

Find the lengths a and b indicated. Write your answers to three decimal places. Be sure to use your wits for these!



Now your students are ready for any standard textbook question on this topic, just by exercising their wits and being agents of their own learning and growth of understanding.

What follows is a sampler of trickier textbook challenges. Don't first explain the jargon that might be foreign to your students. Use these moments as opportunities for students to exercise their agency.

Also, there has been no mention of the tangent function as of yet. This is deliberate.

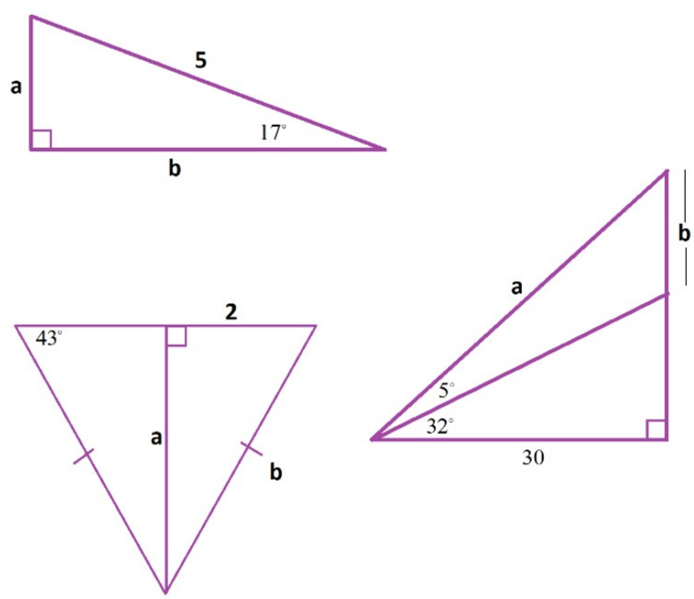
Still Yet More Practice Material

Here are some more problems to try. Let's assume all answers are to be to three-decimal places.

There might be words in some of these questions you have never seen before. Just do your best to use common sense to make good, educated guesses as to their meaning. Usually drawing a picture of the situation at hand will reveal what must be meant. (One can, of course, always just google any unusual terms.)

Some of these problems are tricky. That is okay! Sit with them for a spell, try to make some progress and then let them be and go to some other tasks while you wait for a flash of insight to come to your brain.

Question 1: Find the missing side lengths.



Question 2: As observed at the top of a lighthouse 150 feet about seal level the angle of depression of a ship sailing directly towards the lighthouse changes from 30° to 40°. How far did the ship travel during this period of observation?

Question 3: I am looking out of my tenth-floor apartment window to a building directly opposite my building. The building I see is 50 meters away. Using my protractor, I estimate the angle of elevation to the top of this second building to be 25° and the angle of depression down base of the building as 36°. How tall is that building?

Question 4: (From the 10^{th}-grade Tamil Nadu State Curriculum, India.)
There are two temples, one on each bank of a river just opposite one another. One temple is 40 meters high. As observed from the top of this temple, the angles of depression of the top and the foot of the other temple are 12° and 21°, respectively.

How wide is the river?

How tall is the other temple?

Question 5: At 6 pm the Sun has an angle of elevation of 10.6°. At this time, when I stand tall, my shadow is 32 feet long. To the nearest inch, how tall am I?

You calculator also has the means "undo" sine and cosine values. For instance, if you are wondering which angle θ in a right triangle has sine value 0.8 look for the $\sin^{-1}$ button on your calculator. (The superscript -1 indicates "undoing.") Then type in $\sin^{-1}(0.8)$ to get 53.1°.

Quick Practice 6: Find $\sin(1^{\circ})$, $\sin^{-1}(0.5)$, $\cos\left(\frac{1}{2}^{\circ}\right)$, $\cos^{-1}\left(\frac{1}{2}\right)$.

Question 7: Why does your calculator give an error message for $\sin^{-1}(1.01)$? Should your calculator also give an error message for $\cos^{-1}(1.01)$? (Does it?)

Question 8: What are the three angles in a triangle with side lengths 3, 4, and 5?

Question 9: Prodipta stands at the edge of a cliff on one side of a river, 20 meters wide. The cliff is 30 meters tall. What is the angle of depression of his line of site from the top of the cliff to the river-bank directly opposite him?

Question 10: What is the angle of elevation of the graph of the line $y = 2x + 5$?

Question 11: A straight section of road has a 10% grade.
a) What is the angle of elevation of that section of road?
b) If I drive 50 meters upwards on that section of road, how high have I ascended?

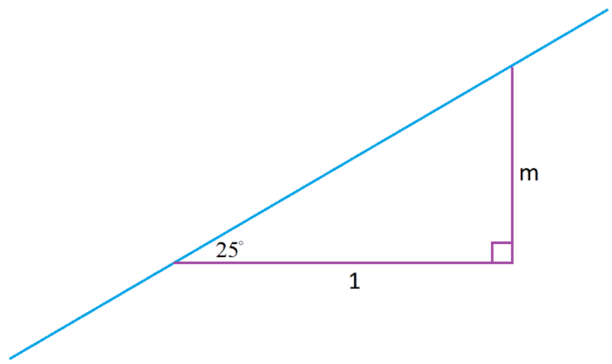
WHERE'S THE TANGENT FUNCTION?

The tangent function is not actually needed in any of the standard trigonometry work. But it is useful as a shortcut to repeated work.

Perhaps have the following discussion with your students.

Consider the following problem: *What is the slope of a line with angle of elevation* 25° *?*

To answer this, recall that the slope m of a line is "the rise needed to get back onto the line if one takes a unit step horizontally off of the line." We thus have this picture.



We see a right triangle. If we label its hypotenuse r, then we have

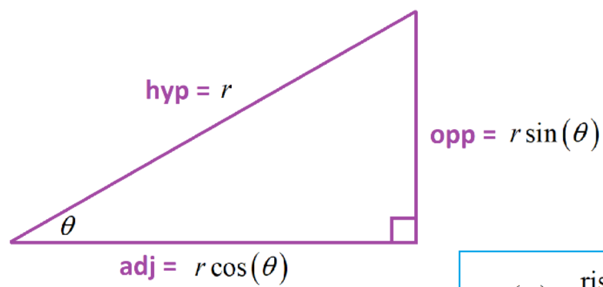
$$m = r\sin\left(25^{\circ}\right)$$
$$1 = r\cos\left(25^{\circ}\right)$$

Solving, we get $m = \dfrac{\sin\left(25^{\circ}\right)}{\cos\left(25^{\circ}\right)} \approx 0.466$.

In many practical applications it is important to know the slopes of lines and line segments, and so people have focused on giving the ratio "sine over cosine" its own name. It is called **tangent**.

We set $\tan\left(\theta\right) = \dfrac{\sin\left(\theta\right)}{\cos\left(\theta\right)}$.

SOH, CAH, and then TOA



$$\boxed{\tan(\theta) = \frac{\text{rise}}{\text{run}} = \frac{\text{opp}}{\text{adj}} = \frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Calculators have a $\tan$ button, as well as a $\tan^{-1}$ button to "undo" a tangent value.

For example,

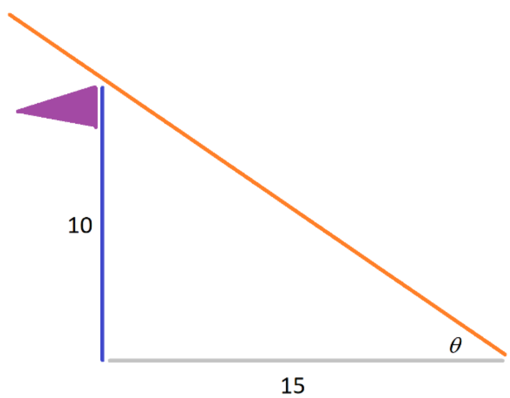
$\tan(25^\circ) \approx 0.466$ and a line with angle of elevation 25° has slope 0.466.

$\tan^{-1}(2) \approx 63^\circ$ and a line of slope 2 has angle of elevation 63°.

Practice Example: *A vertical flagpole 10 meters tall casts a shadow 15 meters long. To the nearest degree, what is the angle of elevation of the Sun at that moment?*

Answer without use of tangent: Let θ be the angle of elevation.

Drawing a diagram, we see a natural right triangle to consider.



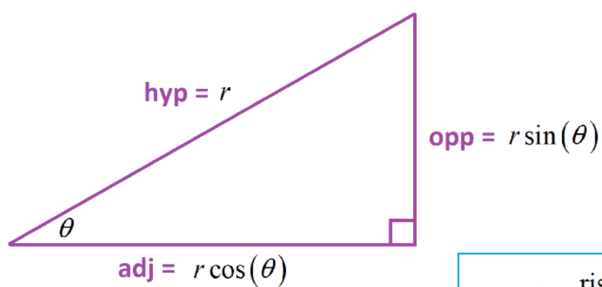
SOH, CAH, and then TOA

By the Pythagorean theorem, the hypotenuse of this triangle is $\sqrt{325}$. From $10=\sqrt{325}\sin(\theta)$ we see $\sin(\theta)=\frac{10}{\sqrt{325}}$, and so $\theta=\sin^{-1}\left(\frac{10}{\sqrt{325}}\right)\approx 33^{\circ}$. (We obtain this same answer if we work with $15=\sqrt{325}\cos(\theta)$ instead.)

Answer using tangent: We have $\tan(\theta)=\text{slope}=\frac{\text{rise}}{\text{run}}=\frac{10}{15}$ and so $\theta=\tan^{-1}\left(\frac{10}{15}\right)\approx 33^{\circ}$.

Indeed, we see that having slope values, that is tangent values, already at hand can simplify work.

Final Practice Exercise



$$\boxed{\tan(\theta)=\frac{\text{rise}}{\text{run}}=\frac{\text{opp}}{\text{adj}}=\frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Which of the 11 questions in the previous problem set can be solved more swiftly making use of the tangent function?

If you are curious about some more history behind the story of trigonometry (how the "tangent" operation got this strange name), perhaps look at this video and its follow-on video here.

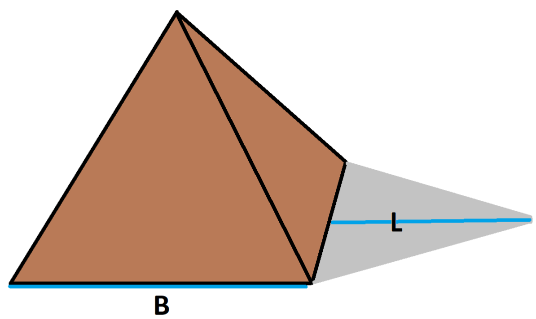
STUDENT HANDOUTS

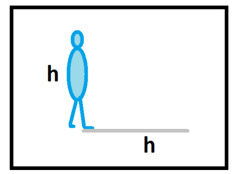
CLASS DISCUSSION ITEM 1

Thales of Ionia (640-546 BCE) garnered tremendous fame as a geometer by deducing the height of the Great Pyramid of Egypt simply by measuring shadow lengths.

He waited for the day of the year that the Sun set directly behind the pyramid, and also waited for the exact time of day that his body cast a shadow the same length as his height. Then, at that very moment, he measured the length L of the shadow shown. As it is easy to measure the base width B of the Great Pyramid, Thales had all the information he needed to deduce its height.

Can you see how? Can you explain the mathematics behind his method?



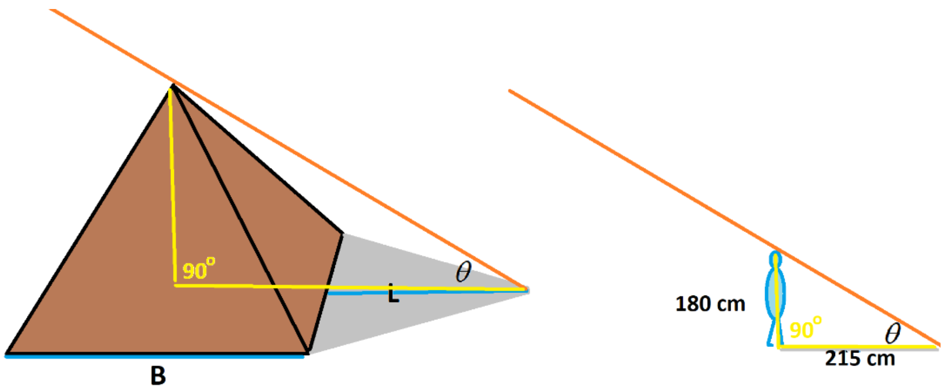


The base length of the pyramid is $B = 230$ m. Suppose the length of the shadow at his special time of the special day is $L = 32$ m. What is the height of the pyramid?

CLASS DISCUSSION ITEM 2

Did Thales really need to wait for the exact time of day when his shadow length matched his height?

Suppose Thales was 180 cm tall and he conducted this exercise at a time of day when his shadow was 215 cm long. Could he still have determined the height of the Great Pyramid?



The base length of the pyramid is $B = 230$ m.

Given that you do already know the height of the pyramid from the previous exercise (147 m), what is the length L of the shadow in this scenario?

How tall is a flagpole?

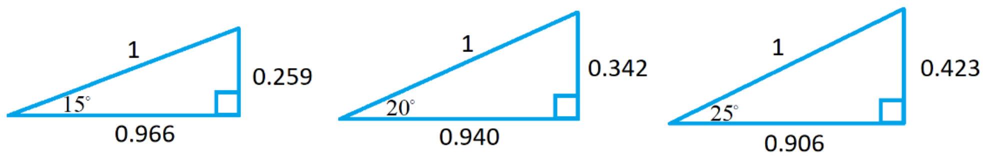
ACTIVITY

On a sunny day head out to a plaza or a field with flagpole equipped with a tape measure and a friend. Have your friend stand tall. Measure the length of her or his shadow. Measure the length of the shadow of the pole.

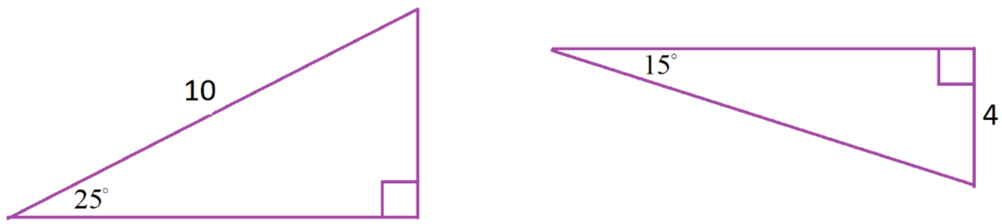
Do you have all the information you need to find the height of the pole? (If not, measure something in addition!)

Some Practice

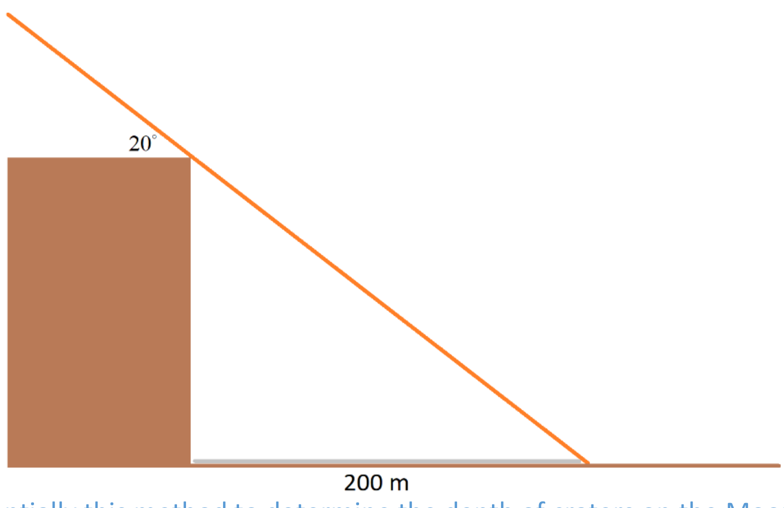
Here the side lengths of three right triangles each with hypotenuse 1. Use them for the following two questions.



Question 1: Find the two missing side lengths in each of these right triangles.

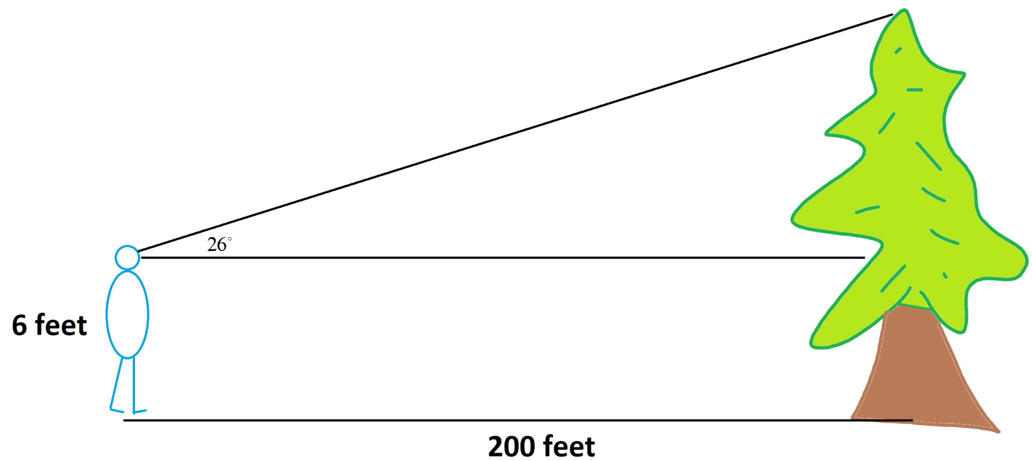


Question 2: A cliff casts a shadow 200 meters long when the rays of Sun hit the surface of the Earth at an angle of 20°. How high is the cliff?



(Scientists use essentially this method to determine the depth of craters on the Moon by looking at the lengths of shadows on surface photographs and knowing the angle of the Sun at the time the photographs were taken.)

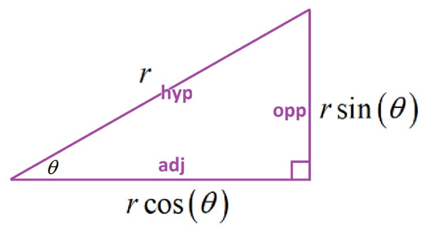
Question 3: Estimate the height of the tree.



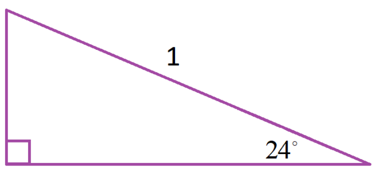
More Practice

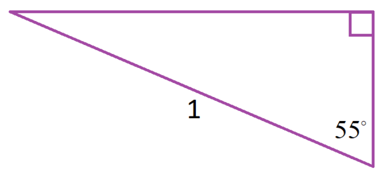
For a right triangle possessing an angle θ we have

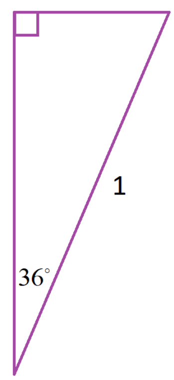
$$hyp = r$$
$$opp = r\sin(\theta)$$
$$adj = r\cos(\theta)$$



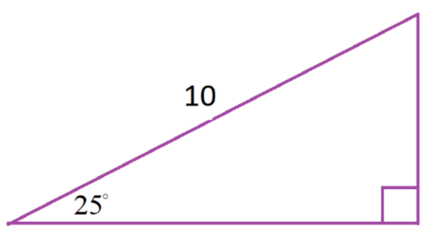
Question 1: According to your calculator, to three decimal places, what are the two missing side lengths in each of these triangles?

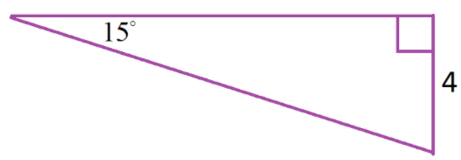






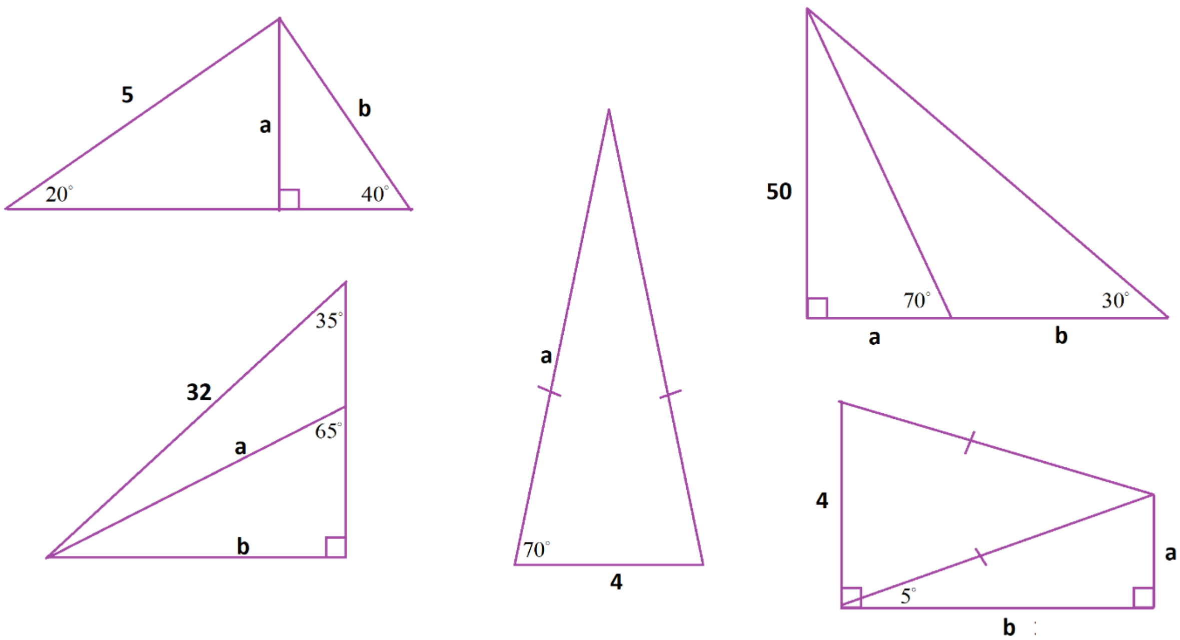
Question 2: Find the two missing side lengths in each of these right triangles.





Yet More Practice

Find the lengths a and b indicated. Write your answers to three decimal places. Be sure to use your wits for these!



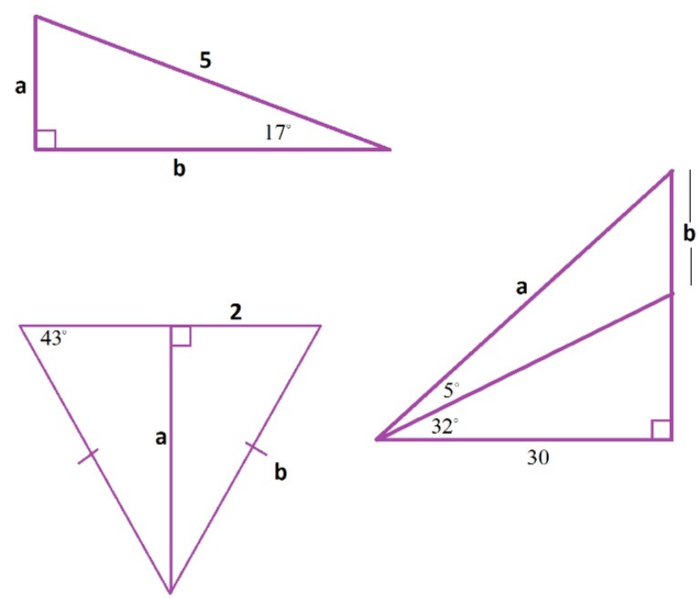
Still Yet More Practice Material

Here are some more problems to try. Let's assume all answers are to be to three-decimal places.

There might be words in some of these questions you have never seen before. Just do your best to use common sense to make good educated guesses as to their meaning. Usually drawing a picture of the situation at hand will reveal what must be meant. (One can, of course, always just google any unusual terms.)

Some of these problems are tricky. That is okay! Sit with them for a spell, try to make some progress and then let them be and go to some other tasks while you wait for a flash of insight to come to your brain.

Question 1: Find the missing side lengths.



Question 2: As observed at the top of a lighthouse 150 feet about seal level the angle of depression of a ship sailing directly towards the lighthouse changes from 30° to 40°. How far did the ship travel during this period of observation?

Question 3: I am looking out of my tenth-floor apartment window to a building directly opposite my building. The building I see is 50 meters away. Using my protractor I estimate the angle of elevation to the top of this second building to be 25° and the angle of depression down base of the building as 36°. How tall is that building?

Question 4: (From the 10^{th}-grade Tamil Nadu State Curriculum, India.)
There are two temples, one on each bank of a river just opposite one another. One temple is 40 meters high. As observed from the top of this temple, the angles of depression of the top and the foot of the other temple are 12° and 21°, respectively.

How wide is the river?

How tall is the other temple?

Question 5: At 6 pm the Sun has an angle of elevation of 10.6°. At this time, when I stand tall, my shadow is 32 feet long. To the nearest inch, how tall am I?

You calculator also has the means "undo" sine and cosine values. For instance, if you are wondering which angle θ in a right triangle has sine value 0.8 look for the $\sin^{-1}$ button on your calculator. (The superscript -1 indicates "undoing.") Then type in $\sin^{-1}(0.8)$ to get 53.1°.

Quick Practice 6: Find $\sin(1^\circ)$, $\sin^{-1}(0.5)$, $\cos\left(\frac{1^\circ}{2}\right)$, $\cos^{-1}\left(\frac{1}{2}\right)$.

Question 7: Why does your calculator give an error message for $\sin^{-1}(1.01)$? Should your calculator also give an error message for $\cos^{-1}(1.01)$? (Does it?)

Question 8: What are the three angles in a triangle with side lengths 3, 4, and 5?

Question 9: Prodipta stands at the edge of a cliff on one side of a river, 20 meters wide. The cliff is 30 meters tall. What is the angle of depression of his line of site from the top of the cliff to the river bank directly opposite him?

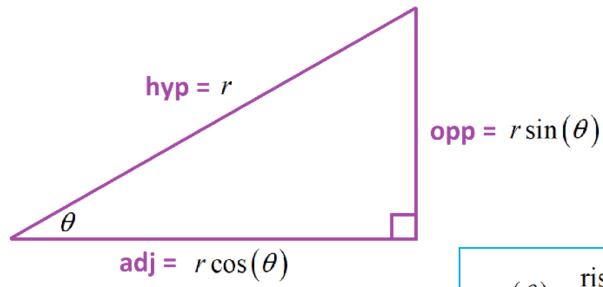
Question 10: What is the angle of elevation of the graph of the line $y = 2x + 5$?

Question 11: A straight section of road has a 10% grade.

c) What is the angle of elevation of that section of road?
d) If I drive 50 meters upwards on that section of road, how high have I ascended?

SOH, CAH, and then TOA

Final Practice Exercise



$$\boxed{\tan(\theta) = \frac{\text{rise}}{\text{run}} = \frac{\text{opp}}{\text{adj}} = \frac{\cancel{r}\sin(\theta)}{\cancel{r}\cos(\theta)}}$$

Which of the 11 questions in the previous problem set can be solved more swiftly making use of the tangent function?

BRIEF SOLUTIONS

Some Practice

Question 1:

* The triangle with side of length **10.**
We have a scale factor of 10.
The remaining two sides are $0.423 \times 10 = 4.23$ and $0.906 \times 10 = 9.06$.

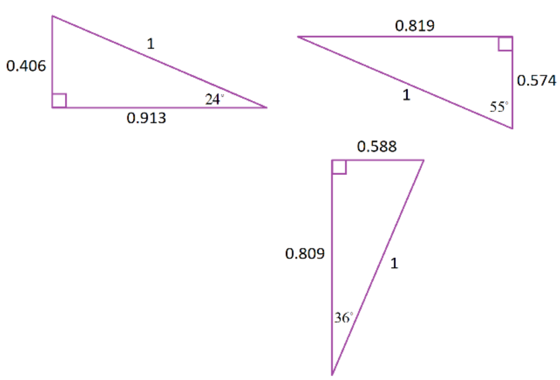
* The triangle with side of length **4.**
Scale factor is given by $4 = 0.259r$ and so $r = 15.444$. The remaining two sides are $0.966 \times 15.444 = 14.919$ and $1 \times 15.444 = 15.444$.

Question 2: We see a right triangle with angle 20°. Let r be the scale factor between this triangle and the basic one given. We have $200 = r \times 0.940$ and so $r = \dfrac{200}{\cos\left(20^\circ\right)} \approx 212.766$. Thus the height of the cliff is $r \times 0.342 \approx 73$ meters.

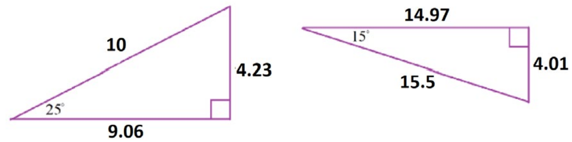
Question 3: We see a right triangle with angle approximately 25°. So $200 = r \times 0.906$ for some scale value r. We have $r \approx 220.751$. Thus the height of the tree is approximately $6 + 0.423 \times 220.751 \approx 102$ feet.

More Practice

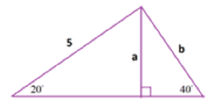
Question 1



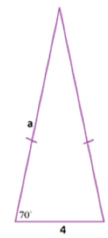
Question 2:



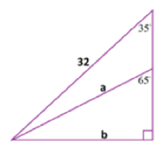
Yet More Practice



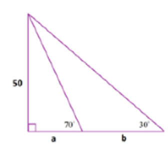
$a = 1.710$
$b = 2.660$



2 = a cos(70)
a = 5.848



$b = 32\sin(35) = 18.354$
$18.354 = a\sin(65)$
$a = 20.252$



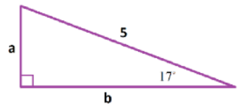
50 = r x sin(70). So r = 53.209.
a = r x cos(70) = 18.199

50 = R x sin(30). So R = 100.
a+b = R x cos(30) = 86.603
b = 86.603 - a = 68.404

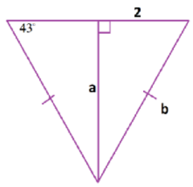
We see that a = 2.
Let r be the hypotenuse of the right triangle.
2 = r x sin(5) and so r = 22.947,
b = r x cos(5) = 22.860.

Still Yet More Practice

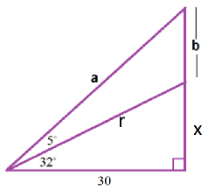
Question 1:



a =5sin(17) = 1.462
b = 5cos(17) = 4.782



2 = bcos(43), so b = 2.735.
a = bsin(43) = 1.865

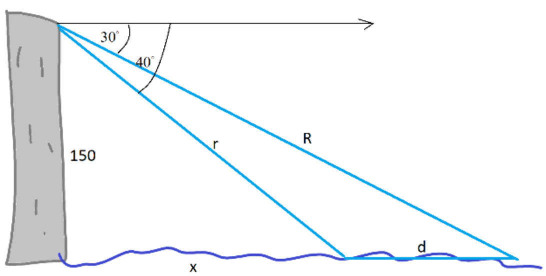


30 = a cos(37), so a = 37.564.
b+x = asin(37) = 22.607

30 = rcos(32), so r = 35.375.
x = rsin(32) = 18.746.

b = 22.607 - 18.746 = 3.861

Question 2:
Label a diagram as shown.



$150 = r\sin(40)$ and so $r = 233.359$.

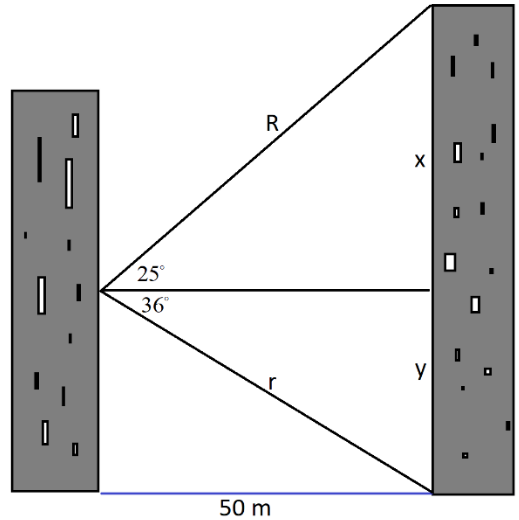
$150 = R\sin(30)$ and so $R = 300$.

$x = r\cos(40) = 178.763$.

$x + d = R\cos(30) = 259.808$.

So $d = 81.045$ feet.

Question 3:
Label a diagram as shown.

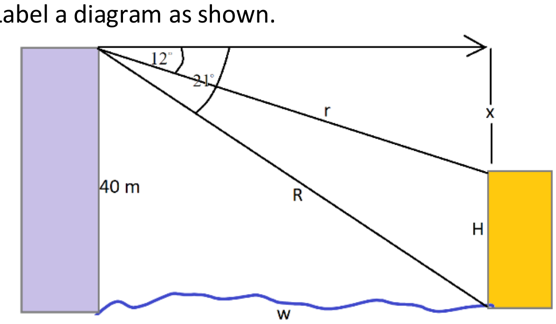


$50 = r\cos(36)$ and so $r = 61.803$.

$50 = R\cos(25)$ and so $R = 55.169$.

$x + y = r\sin(36) + R\sin(25) = 59.642$ meters.

Question 4:
Label a diagram as shown.



$40 = R\sin(21)$ and so $R = 111.617$.

$w = R\cos(21) = 104.204$ meters.

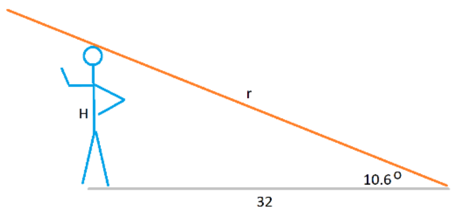
$w = r\cos(12)$ and so $r = 106.532$.

$x = r\sin(12) = 22.149$.

$H + x = 40$.

$H = 40 - x = 17.851$ meters.

Question 5:
Label a diagram as shown.



$32 = r\cos(10.6)$ and so $r = 32.556$.
$H = r\sin(10.6) = 5.989$ feet.

Question 6:
$\sin(1^\circ) \approx 0.017$, $\sin^{-1}(0.5) = 30^\circ$.

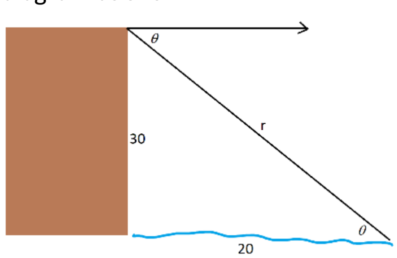
$\cos\left(\frac{1}{2}^\circ\right) \approx 0.99996$, $\cos^{-1}\left(\frac{1}{2}\right) = 60^\circ$.

Question 7: The hypotenuse is the longest side of a right triangle. (Reason: If a right triangle has sides a and b, then its hypotenuse is $\sqrt{a^2+b^2}$. This is larger than $\sqrt{a^2+0^2} = a$ and $\sqrt{0^2+b^2} = b$.)

There is no right triangle of hypotenuse 1 with an opposite side or adjacent side of length 1.01.

Question 8: This is a right triangle, so one angle has measure 90°.
Let θ be the angle opposite the side of length 3.
Then $3 = 5\sin(\theta)$ and so $\sin(\theta) = 0.6$ giving $\theta = \sin^{-1}(0.6) \approx 36.870^\circ$. The other angle in the triangle is the complement to this: 53.130°.

Question 9:
Label a diagram as shown.

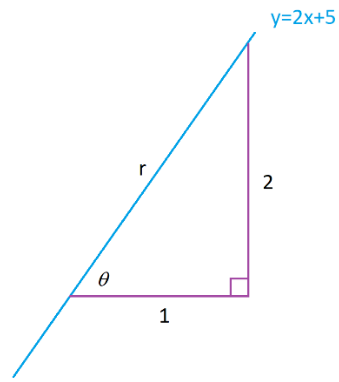


By the Pythagorean Theorem, $r = \sqrt{1300}$.

We have $30 = r\sin(\theta)$ and so

$$\theta = \sin^{-1}\left(\frac{30}{\sqrt{1300}}\right) \approx 56.310^\circ.$$

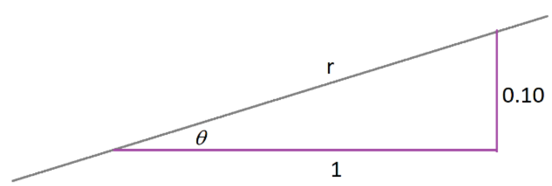
Question 10: We have a line of slope 2.
Label a diagram as shown.



Here $r = \sqrt{5}$ and $2 = \sqrt{5}\sin(\theta)$. Thus

$$\theta = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right) \approx 63.435^\circ$$

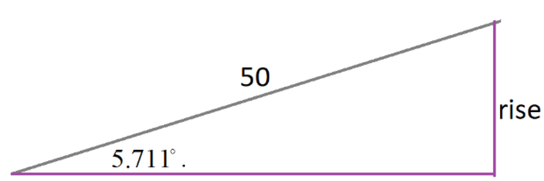
Question 11:
a) Label a diagram as shown.



We have $r = \sqrt{1.01}$ and

$$\theta = \sin^{-1}\left(\frac{0.10}{\sqrt{1.01}}\right) \approx 5.711^\circ.$$

b) $rise = 50\sin(5.711) = 4.975$ meters.



Final Practice Exercise

Elements of question 1, and questions 2, 3, 4 5, 8, 9, 10, and 11 of the previous problem set are more swiftly solved using the tangent operation.