

Logarithms for Humans

PART 3

Now that we know what logarithms are ...



Logarithms are powers.

$$b^x = N$$

x is the power of b that gives the answer N

$$x = \log_b(N)$$

Example: Please rewrite $7^3 = 343$ as a logarithmic statement.

Answer: We see that 3 is the power of 7 giving the answer 343. Consequently:

$$3 = \log_7(343)$$

Example: Please rewrite $\log_{0.5}(16) = -4$ as a direct statement about exponents.

Answer: We are being told that -4 is the power of 0.5 that give this the value 16. That is,

$$(0.5)^{-4} = 16$$

Question 1: Show that $(0.5)^{-4} = 16$ is a true statement!

Question 2: Rewrite $\log_{0.1}(10) = -1$ as a statement about powers.

Question 3: Give an example of two positive numbers b and N such that

$$\log_b(N) = -3$$



Schoolbooks are chock full of icky-looking practice examples. Why such examples would ever appear in real life beats me!

Example: Please evaluate $\log_{\frac{1}{a^2}} \left(a^{\frac{2}{3}} \right)$.

First Answer: No!

Second Answer: Oh. Okay.

Deep breath.

We're looking for the power of $\frac{1}{a^2}$ that gives $a^{\frac{2}{3}}$.

That is, we are looking for the value x so that

$$\left(\frac{1}{a^2} \right)^x = a^{\frac{2}{3}}$$

Following all our rules of exponents, this reads as:

$$a^{-2x} = a^{\frac{2}{3}}$$

We can now see that $x = -\frac{1}{3}$ does the trick.

So,

$$\log_{\frac{1}{a^2}} \left(a^{\frac{2}{3}} \right) = -\frac{1}{3}$$

Question 4: Please evaluate $\log_{(b^{0.5})} \left(\frac{1}{\sqrt{b}} \right)$.

(Feel free to say NO!)



What's more fun are questions that are like logic puzzles.

Question 5:

a) What is the value of $\log_{56}(56)$?

b) In general, what can you say about the value of

$$\log_b(b)$$

for a positive number b ?

Question 6:

a) What is the value of $\log_7(1)$?

b) In general, what can you say about the value of

$$\log_b(1)$$

for a positive number b ?

Now things get tautological!

Question 7: Who is Lucy's daughter's mother?

Question 8:

a) What is the value of $\log_{11}(11^{4505})$?

b) In general, what can you say about the value of

$$\log_b(b^x)$$

for a positive number b and real number x ?



Question 9:

a) What is the value of $9^{\log_9(777)}$?

b) In general, what can you say about the value of

$$b^{\log_b(N)}$$

for positive numbers b and N ?

Playing with the definition of $\log_b(N)$ for positive numbers b and N is all well and good, but two deep questions about them are hanging over our heads.

- 1. Who cares about logarithms today?**
2. Even if I imagine that I live in the 1600s and care about logarithms, what has our definition of logarithms got to do with Napier's work from last lesson?
How do logarithms convert multiplication problems into addition problems?

We shall attend to both questions.



If you'd like to practice the mechanics of logarithms, try these textbook problems.

Practice 10 Please rewrite each of these statements in logarithmic form.

a) $3^5 = 243$ b) $625^{\frac{1}{4}} = 5$ c) $2^0 = 1$

Practice 11 Please rewrite each of these statements as a statement about powers.

a) $2 = \log_7(49)$ b) $\frac{1}{2} = \log_5(\sqrt{5})$ c) $a = \log_3(b)$

Practice 12 Kindly evaluate each of these expressions.

a) $\log_2(8^{303})$ b) $\log_{\frac{1}{2}}\left(\frac{1}{16}\right)$ c) $\log_m((m^3)^2)$

Practice 13 If you are still here doing practice problems ... For each statement, kindly find the value of b that makes the statement true.

a) $\log_b(125) = 3$ b) $\log_b(125) = -3$ c) $\log_b(8) = -0.25$

Practice 14 Since you have come this far ... For each statement, please consider finding a value of x that makes the statement true.

a) $\log_3(2x + 1) = 4$ b) $\log_{10}(x + 2) = -2$ c) $\log_x(4) = x$