Logarithms for Humans

PART 1:

What made people want them?
What problem were people trying to solve?

A Scientific Crisis – and its Resolution

There was a scientific crisis in Europe during the Renaissance, brought on simply by the task of having to conduct arithmetic by hand.

The 1400s and 1500s saw Western scholars make new advances in the arts and sciences, leading to new understandings of the natural world. The invention of Galileo's telescope, which he called a *perspicillum*, opened up the workings of the heavens too to fuel extraordinarily rapid progress in astronomy.

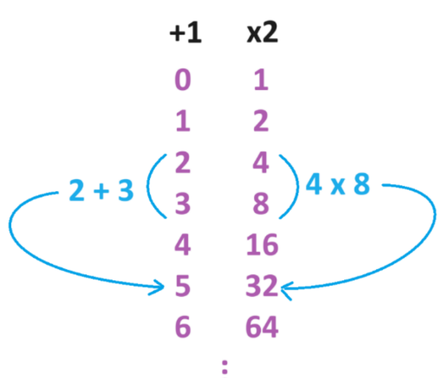
As data gathering methods became more precise, astronomers found themselves burdened by the simple process of arithmetic when performing calculations. While addition was manageable with chalk on slates or with quill on paper, conducting multiplication was onerous.

278,862,108	278,862,108
+ 305,721,661	x 305,721,661
=	=
Not Too Bad	**ICK!**

Astronomers used extensive tables of angular measures in their work with numbers adjusted by a factor of 10,000,000 to avoid dealing with fractions (decimals and decimal notation was not yet in use). This left them regularly conducting calculations with seven-, eight-, nine-digit numbers and they were severely hampered when it came to multiplying and dividing numbers or finding their square and cube roots.

Progress in science was severely held back.

Yet mathematicians had observed an interplay between multiplication and simpler addition. For example, in 1544 German mathematician Michael Stifel illustrated such a connection with the counting numbers (constructed by repeatedly adding 1 to zero) and the doubling numbers (constructed by repeatedly multiplying the number 1 by two).



The trouble was that scholars did not have a clear understanding of exponents beyond whole-number exponents. This meant that they could not, for instance, provide values between the lines in Stifel's table and conduct a wider range of products.

In the 1590s, Scottish mathematician John Napier decided to tackle this very problem. He later wrote in reflection in 1614:

> *Seeing there is nothing that is so troublesome to mathematical practice, nor that doth more molest and hinder calculators, than the multiplications, divisions, square and cubical extractions of great numbers ... I began therefore to consider in my mind by what certain and ready art I might remove those hindrances.*

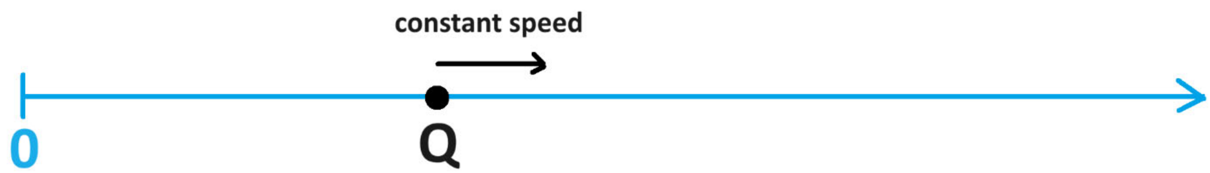
He succeeded. Napier's approach might seem strange and curious from our modern perspective. But we must remember that an understanding of exponents (yet alone a notation for them) was not available to him. He had to be innovative.

Napier took a kinematic approach. He imagined a particle P moving along a line segment of length $r = 10{,}000{,}000$, starting at 0 on the left and moving to the right.

The particle starts out at a speed of r units per second, but it gets slower and slower as it moves to the right. In fact, the speed of particle is given by how much further it needs to travel: if the particle has x more units to cover, it now moves at a speed of x units per second.

Question 1
Will the particle ever reach the rightmost point of the segment?

Next, Napier imagined a second particle Q moving along a line, starting at 0 on the left, but always moving the constant speed of r units per second. This particle will move infinitely far to the right.



Napier was able to show that distance particle P has yet to travel and the distance Q has already traveled have the same sort of multiplicative/additive relationship akin the one Stifel had pointed out with the doubling numbers and the counting numbers.

Moreover, Napier had figured out a computational method that allowed him to approximate the distances covered by the particles every thousandth of a second, which gave him a table like Stifel's, but with the "in-between" numbers.

And it was this table that saved science! He showed the world how to reduce complicated arithmetical operations to ones of manageable addition and subtraction.

Napier coined the name **logarithm** for the numbers in his table, coming from the Greek words *logos* (ratio) and *arithmos* (number) to represent how he was comparing the values of the distances in the motions of the two particles.

Napier published his 90-page table of logarithm values in 1614.

It wasn't until more than a century that mathematicians properly understood exponents and could see Napier's work as theory of general exponents. But by then the name *logarithm* was firmly entrenched in scientific community's vernacular, and the name stays with us to this day.

And that's a fine thing. We should continue to honor Napier's magnificent achievement that saved scientific progress at a crucial time.

Logarithm Tables

Using my modern understanding of exponents (and my calculator!), I am able to do what Stifel and Napier could not initially do, namely, accurately fill in some values between the lines of Stifel's table.

I will also call these values *logarithms*, but they are not the logarithmic values Napier computed and what is shown here is not part of the table he presented to the world. But we can use this table to illustrate, nonetheless, how scholars converted multiplication problems into addition problems via Napier's work.

number	Stifel logarithm
1	0
2	1
3	1.585
4	2
5	2.322
6	2.585
7	2.807
8	3
9	3.170
10	3.322
11	3.459
12	3.585
13	3.700
14	3.807
15	3.907
16	4

Do you see the doubling numbers (this time to the left) and the matching counting numbers to the right, as per Stifel's table?

The decimal values you see have all been rounded to three decimal places. (So, any discrepancies we might encounter will be because of this rounding in the thousandth place. But this should not be too problematic for us.)

Also, I've only given logarithm values for the first sixteen numbers, so the multiplication problems we'll illustrate here will be small!

Suppose we wanted to work our 3×5 using the table. (Of course, we know the answer is 15.)

Here's the method:

1. The logarithmic value for $\mathbf{3}$ is 1.585.
2. The logarithmic value for $\mathbf{5}$ is 2.322.
3. The sum of these two values is 3.907.
4. Looking back at the table, the number with logarithmic value 3.907 is $\mathbf{15}$.

$$\begin{array}{r} 3 \\ \times\ 5 \\ \hline = \end{array} \qquad\qquad \begin{array}{r} 1.585 \\ +\ 2.322 \\ \hline =\ 3.907 \end{array}$$

In summary …

To multiply two numbers: Look up their logarithmic values. Add those values and then see which number has logarithmic value equal to the sum. That number is the product of the original two numbers.

Question 2
Use the table to compute each of these products.

3×4 $\qquad$ 7×2 $\qquad$ 3×3

Question 3
a) Knowing that $18 = 3 \times 6$, what is the logarithmic value that goes with the number 18?
b) Do you get the same value you think of 18 as 2×9 instead?
c) Make a guess as to logarithmic value that goes with the number 17.

Question 4
Might you care to research the details of Napier's "ratio of distances" approach to create logarithmic tables that solved the multiplication problem for science?