Logarithms for Humans

PART 6

Practical Example:
Compounding Growth

The human population is growing.

According to an internet search at the time of me writing this, we seem to be averaging about 17.3 births for every 1000 people each year, and 8.9 deaths for every 1000 people each year.

So, we're growing at a rate of an increase of 8.4 additional Earth inhabitants for every 1000 people per annum.

Queston 1 These figures are continually changing. What are global birth and mortality rates at the time of you reading this?

Can you find some data showing how these rates seem to have changed over the decades?

Comment: The data I found is presented in terms of average counts per 1000 people. Some data might be presented instead as a percentage, an average count per 100 people.

For example, the fraction $\frac{17.3}{1000}$ is the same as $\frac{1.73}{100} = 1.73\%$, so our data corresponds to a birth rate of 1.73%.

Just to be clear, we're speaking of growth *rates*, not absolute figures. The total numbers of actual births and deaths depends on the total number of people there are. We've just been told, for every group of 1000, we can expect an average number of 17.3 births, and so on.

For instance, my internet search also tells me there are currently about 8 billion people on the planet. If we trust the figures I've shared, over the coming year there will be

$$8{,}000{,}000{,}000 \times \frac{17.3}{1000} = 138{,}400{,}000 \text{ babies born}$$

$$8{,}000{,}000{,}000 \times \frac{8.9}{1000} = 71{,}200{,}000 \text{ deaths}$$

for a total increase of $67{,}200{,}000$ inhabitants (which is indeed 0.84% of $8{,}000{,}000{,}000$).

We're thus predicting a new human population figure of $8{,}067{,}000{,}000$ by the end of one year.

Question 2 Suppose the birth rate of 1.73% and death rate of 0.89% stay the same for next few years. What do you predict then for the Earth's population after a second year? We'll grow from 8.067 billion humans to how many?

And after a third year? A fourth?

We're seeing that if the Earth's population is a count of P people, then after a year there will be

$P \times 0.0173$ babies born

$P \times 0.0089$ deaths

for an increase $P \times 0.0173 - P \times 0.0089 = P \times 0.0084$ in the count of people on the planet.

Over a year, the human population changes from P people to $P + P \times 0.0084$ people.
We have

$$P \;+\; P \times 0.0084 = P \times (1 + 0.0084)$$
$$= P \times 1.0084$$

According to our overly basic analysis, the Earth's population changes by a factor of $\mathbf{1.084}$ from year to year.

Let's play with this.

We started with a population of $P = 8{,}000{,}000{,}000$ people.

After 1 year the population grows to

$$P_1 = 8{,}000{,}000{,}000 \times 1.084 = 8{,}067{,}000{,}000$$

After another year, by year 2, the population grows to

$$P_2 = 8{,}000{,}000{,}000 \times 1.084 \times 1.084 = 8{,}744{,}628{,}000$$

By year 3 the population is

$$P_3 = 8{,}000{,}000{,}000 \times 1.084 \times 1.084 \times 1.084 = 9{,}479{,}176{,}752$$

By year 4 the population is

$$P_4 = 8{,}000{,}000{,}000 \times 1.084 \times 1.084 \times 1.084 \times 1.084 = 11{,}046{,}052{,}825$$

If P_n denotes the population by the end of year n, then our (likely unrealistic) reasoning predicts that the Earth's population will be given by

$$P_n = 8{,}000{,}000{,}000 \times (1.084)^n$$

Question 3 Let's continue to believe that the Earth's population will grow without bound at a constant rate of 0.84% per annum.

According to our model:

a) What will the Earth's population by the end of ten years?

b) In how many years will the Earth's population be one-hundred billion?

Question 4 Assume that the birth and death rates of fruit flies living in my compost bin are each constant. Yesterday there were 160 fruit flies in the bin. Today there are 200.

Create simple mathematical model that predicts the number of fruit flies I can expect living in my compost bin each day.

On which day will I have just over a million fruit flies according to your model?

Question 5 There are fruit flies in my recycling bin too, but the population of them there is not thriving. I've observed they are breeding at a constant birth rate of 10% per day, but dying at a rate of 12% per day.

There are currently 100 fruit flies in my recycling bin.

Create simple mathematical model that predicts the number of fruit flies I can expect living in my recycling bin each day.

According to your model, on which day will I have just one lonely fruit fly in my bin?