Logarithms for Humans

A TYPICAL (unimaginative) EXAM

Here is an example of a math exam with questions typical for many curriculums. It is designed to resemble the style, tone, and structure you might encounter in a formal classroom setting.

A Few Notes to Keep in Mind:

Authoritative Tone:

This exam uses a formal, commanding tone. There’s no use of words like “please,” “kindly,” or “if you will.” While this can seem harsh, it’s a common style in mathematics exams. Try not to let it unsettle you—it’s just the convention!

Ambiguous Terms:

Words such as *simplify* and *expand* often appear in exams, even though their meanings can be subjective. What seems “simple” in one context might feel awkward in another. As you work through the exam, make your best effort to interpret what the examiner is asking for.

Unfamiliar Material:

It’s possible the exam might include a question or concept that feels unfamiliar or wasn’t explicitly taught. Don’t panic! Take a deep breath and use the context of the question, along with your mathematical reasoning, to deduce what’s being asked. You have the tools and knowledge to approach any question thoughtfully.

Logarithms Exam

Time: 60 minutes
Total Marks: 50

Section A: Basic Logarithms (10 marks)

1. (2 marks) Evaluate the following logarithms:

   a) $\log_2(16)$
   b) $\log_5(125)$

2. (2 marks) Rewrite the following expressions in exponential form:

   a) $\log_3(81) = 4$
   b) $\log_{10}(0.01) = -2$

3. (2 marks) Rewrite the following expressions in logarithmic form:

   a) $4^3 = 64$
   b) $10^{-3} = 0.001$

4. (4 marks) Simplify using the properties of logarithms:

   a) $\log_3(3^5)$
   b) $\log_4(80) - \log_4(20)$
   c) $2\log_5(25)$
   d) $\log_7(49 \cdot 7^x)$

Section B: Solving Equations (15 marks)

5. (3 marks) Solve for x:
   a) $\log_2(x) = 6$
   b) $\log_4(x) = \frac{1}{2}$
   c) $10^{x+1} = 1000$

6. (3 marks) Solve for x in the following:
   a) $\log_3(x) + \log_3(4) = 2$
   b) $\log_5(x) - \log_5(2) = 1$
   c) $\log_2(x^2) = 6$

7. (3 marks) Use the **change of base formula** to calculate $\log_6(50)$ to two decimal places. Show your work.

Editorial Comment: It is possible to figure out the answer to this question without having memorized the "change of base formula. (I certainly don't have it in my head!)

8. (6 marks) Expand or condense the following logarithmic expressions:

   a) Expand: $\log_2(16x^3)$
   b) Expand: $\log_5\left(\frac{x^2 y}{z^3}\right)$
   c) Condense: $\frac{1}{2}\log_4(A) - \log_4(B) - \log_4(C)$

Section C: Applications (25 marks)

9. **Richter Scale** (6 marks)

The Richter scale measures the magnitude of an earthquake using the formula:

$$M = \log_{10}\left(\frac{A}{A_0}\right)$$

where A is the amplitude of the earthquake's waves and A_0 is a reference amplitude.

a) If the amplitude of an earthquake is 1000 times the reference amplitude, what is the magnitude M? (2 marks)

b) If an earthquake has a magnitude$M = 5$, how many times larger is its amplitude than the reference amplitude? (4 marks)

10. **Population Growth** (6 marks)

A city's population grows according to the formula:

$$P(t) = P_0 \cdot b^t$$

where P_0 is the initial population, b is the growth factor, and t is time in years.

a) Solve for t in terms of $P(t)$, P_0, and b. (2 marks)
b) If $b = 2$, how many years will it take for the population to triple? (4 marks)

11. **Decibel Scale** (5 marks)

The loudness of a sound is measured in decibels using the formula:

$$L = \log_{10}\left(\frac{I}{I_0}\right)$$

where I is the intensity of the sound and I_0 is the reference intensity.

a) If a sound is 100 times more intense than the reference intensity, what is its decibel level? (2 marks)
b) A sound is measured at 4 decibels. How many times more intense is it compared to the reference intensity? (3 marks)

12. **Cooling Formula** (8 marks)

The temperature of a cooling object is modeled by:

$$T(t) = T_0 \cdot b^t$$

where $T(t)$T is the temperature at time t, T_0 is the initial temperature, and b is the cooling factor.

An object starts at 200°C, cools to 50°C in 2 hours.

a) What is the cooling factor b? (2 marks)
b) How many more hours will it take for the temperature of the object to drop to 10°C?

Answer Key

Section A: Basic Logarithms

1. a) 4 b) 3
2. a) $3^4 = 81$ b) $10^{-2} = 0.01$
3. a) $\log_4(64) = 3$ b) $\log_{10}(0.001) = -3$
4. a) 5 b) 1 c) 4 d) $x + 2$

Section B: Solving Equations

5. a) $x = 64$ b) $x = 2$ c) $x = 2$
6. a) $x = 2.25$ b) $x = 10$ c) $x = 8$ or -8
7. Let's give $\log_6(50)$ the name "G."

$$\log_6(50) = G$$

$$6^G = 50$$

$$\log_{10}(6^G) = \log_{10}(50)$$
$$G \cdot \log_{10}(6) = \log_{10}(50)$$

$$G = \frac{\log_{10}(50)}{\log_{10}(6)} \approx 2.183$$

8. a) $\log_2(16x^3) = \log_2(16) + \log_2(x^3) = 4 + 3\log_2(x)$
   b) $2\log_5(x) + \log_5(y) - 3\log_5(z)$
   c) $\log_4(\frac{\sqrt{A}}{BC})$

Section C: Applications

9. a) $A = 1000A_0$ and so $M = \log_{10}(1000) = 3$
   b) $5 = \log_{10}(\frac{A}{A_0})$ and so $A = 10{,}000 \times A_0$. It is 10,000 times larger.

10. a) We have

$$P(t) = P_0 \cdot b^t$$

$$b^t = \frac{P(t)}{P_0}$$

$$t \log_{10}(b) = \log_{10}\left(\frac{P(t)}{P_0}\right)$$

$$t = \frac{\log_{10}\left(\frac{P(t)}{P_0}\right)}{\log_{10}(b)}$$

a) We want the time for which $P(t) = 3 \cdot P_0$. This occurs at time

$$t = \frac{\log_{10}(3)}{\log_{10}(2)} \approx 1.6$$

The population will triple in about 1 year and 7 months.

11. a) 2 Decibels

b) We have

$$4 = \log_{10}\left(\frac{I}{I_0}\right)$$

and so $\frac{I}{I_0} = 10^4$. It is 10,000 times as intense.

12. a) $b = \frac{1}{2}$

b) Now with an initial temperature of 50°C we need to solve for t in

$$10 = 50 \cdot \left(\frac{1}{2}\right)^t$$

that is,

$$0.2 = (0.5)^t$$

We obtain :

$$t = \frac{\log_{10}(0.2)}{\log_{10}(0.5)} \approx 2.3$$

It reaches 10°C in a further 2.3 hours.